

Universität Konstanz

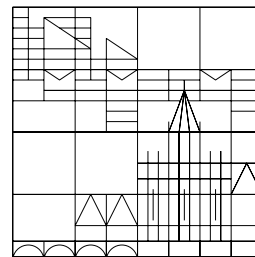
Fachbereich

Mathematik und Statistik

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## Real Algebraic Geometry II

### Exercise Sheet 1

#### Exercise 1

Let  $(M_1, v_1)$ ,  $(M_2, v_2)$  be two valued models with value sets  $\Gamma_1$  and  $\Gamma_2$  respectively. Assume  $(M_1, v_1) \cong (M_2, v_2)$  and  $h : M_1 \rightarrow M_2$  is an isomorphism which preserves the valuation.

- (a) Show that  $\tilde{h} : \Gamma_1 \rightarrow \Gamma_2$  defined by  $\tilde{h}(v_1(x)) := v_2(h(x))$  is a well defined map and an isomorphism of totally ordered sets.
- (b) Show that for  $\gamma \in \Gamma_1$  the map

$$\begin{aligned} h_\gamma : B_1(\gamma) &\rightarrow B_2(\tilde{h}(\gamma)) \\ \pi^{M_1}(\gamma, x) &\mapsto \pi^{M_2}(\tilde{h}(\gamma), h(x)) \end{aligned}$$

is well defined and an isomorphism of modules.

- (c) Show that the skeleton is an isomorphism invariant, i.e. if  $(M_1, v_1) \cong (M_2, v_2)$ , then  $S(M_1) \cong S(M_2)$ .

#### Exercise 2

Let  $[\Gamma, \{B(\gamma) : \gamma \in \Gamma\}]$  be a system of torsion free modules.

- (a) Show that  $\coprod_{\gamma \in \Gamma} B(\gamma)$  is a valued submodule of  $H_{\gamma \in \Gamma} B(\gamma)$ .
- (b) Show that

$$\begin{aligned} S\left(\coprod_{\gamma \in \Gamma} B(\gamma)\right) &\cong [\Gamma, \{B(\gamma) : \gamma \in \Gamma\}] \\ &\cong S(H_{\gamma \in \Gamma} B(\gamma)). \end{aligned}$$

### Exercise 3

Let  $(A, \leq_A)$  and  $(B, \leq_B)$  be two ordered sets.

- (a) Show that if  $A$  and  $B$  are well-ordered, then so is  $A + B$ .
- (b) Show that the sum of ordinals is well-defined and not commutative.  
**Hint:** Compute  $1 + \omega$  and  $\omega + 1$ .
- (c) For any  $n \in \mathbb{N}$  give an example of a subset  $Q_n$  of  $\mathbb{Q}$  such that  $\text{ot}(Q_n) = \omega \cdot n$ .
- (d) Show that if  $A$  and  $B$  are well-ordered, then so is  $A \cdot B$ .
- (e) Show that the product of ordinals is well-defined and not commutative.  
**Hint:** Compute  $2 \cdot \omega$  and  $\omega \cdot 2$ .
- (f) For any  $n \in \mathbb{N}$  give an example of a subset  $Q_n$  of  $\mathbb{Q}$  such that  $\text{ot}(Q_n) = \omega^n$ .

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The exercise will be collected **Thursday, 23/04/2015** until 10.00 at the box near F 441.

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<http://www.math.uni-konstanz.de/~dupont/rag.htm>