Universität Konstanz

Fachbereich
Mathematik und Statistik

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Real Algebraic Geometry II
Exercise Sheet 1

Exercise 1
Let (M_1, v_1) , (M_2, v_2) be two valued models with value sets Γ_1 and Γ_2 respectively. Assume $(M_1, v_1) \cong (M_2, v_2)$ and $h : M_1 \to M_2$ is an isomorphism which preserves the valuation.

(a) Show that $\widetilde{h} : \Gamma_1 \to \Gamma_2$ defined by $\widetilde{h}\big(v_1(x)\big) := v_2\big(h(x)\big)$ is a well defined map and an isomorphism of totally ordered sets.

(b) Show that for $\gamma \in \Gamma_1$ the map

$$\begin{aligned} h_\gamma : B_1(\gamma) &\to B_2\left(\widetilde{h}(\gamma)\right) \\ \pi^{M_1}(\gamma, x) &\mapsto \pi^{M_2}\left(\widetilde{h}(\gamma), h(x)\right) \end{aligned}$$

is well defined and an isomorphism of modules.

(c) Show that the skeleton is an isomorphism invariant, i.e. if $(M_1, v_1) \cong (M_2, v_2)$, then $S(M_1) \cong S(M_2)$.

Exercise 2
Let $[\Gamma, \{B(\gamma) : \gamma \in \Gamma\}]$be a system of torsion free modules.

(a) Show that $\coprod_{\gamma\in\Gamma} B(\gamma)$is a valued submodule of $H_{\gamma\in\Gamma} B(\gamma)$.

(b) Show that

$$\begin{aligned} S\left(\coprod_{\gamma\in\Gamma} B(\gamma)\right) &\cong [\Gamma, \{B(\gamma) : \gamma \in \Gamma\}] \\ &\cong S\left(H_{\gamma\in\Gamma} B(\gamma)\right). \end{aligned}$$

Exercise 3
Let $(A, \leq_A)$and $(B, \leq_B)$be two ordered sets.

(a) Show that if $A$and $B$are well-ordered, then so is $A + B$.

(b) Show that the sum of ordinals is well-defined and not commutative.
Hint: Compute $1 + \omega$and $\omega + 1$.

(c) For any $n \in \mathbb{N}$give an example of a subset $Q_n$of $\mathbb{Q}$such that $\mathrm{ot}\,(Q_n) = \omega.n$.

(d) Show that if $A$and $B$are well-ordered, then so is $A.B$.

(e) Show that the product of ordinals is well-defined and not commutative.
Hint: Compute $2.\omega$and $\omega.2$.

(f) For any $n \in \mathbb{N}$give an example of a subset $Q_n$of $\mathbb{Q}$such that $\mathrm{ot}\,(Q_n) = \omega^n$.

The exercise will be collected **Thursday, 23/04/2015** until 10.00 at the box near F 441.
