French-German Summer School
Galois Theory and Number Theory
Konstanz, 18–24 July 2015

Plan and abstracts of the talks

Tuesday, July 21, 9:00, room A704.

Varieties of Hilbert type

Arno Fehm (Universität Konstanz)

Abstract: Varieties of Hilbert type were introduced in the context of the Noether program. I will discuss classical and recent results on varieties of Hilbert type and address the question of Serre whether a product of two varieties of Hilbert type is again of Hilbert type. As an application I will discuss specializations of covers of algebraic groups.

Tuesday, July 21, 10:30, room A704.

On the Malle Conjecture and the Self-Twisted Cover

Pierre Dèbes (Université de Lille)

Abstract: The Malle conjecture predicts that the number of Galois extensions of $\mathbb{Q}$ of given group G and with discriminant bounded from above by a real number $y > 0$ grows at least like y^a, for some specific exponent $a > 0$. This is known for nilpotent groups and for a few other groups. I will present a method that proves it for S_n, A_n, many simple groups, and more generally for all regular Galois groups over $\mathbb{Q}$. The constructed Galois extensions can further be prescribed some notable local conditions. The method uses a new version of Hilbert's irreducibility theorem that counts the specialized extensions and not just the specialization points. An important ingredient is the self-twisted cover that we will introduce.

Tuesday, July 21, 11:45, room A704.

Statistical number theory in function fields - Sums of two squares

Lior Bary-Soroker (Tel Aviv University)

Abstract: The starting point of this talk is Landaus theorem: the number of sums of two squares up to x is about $K\frac{x}{\log x}$ where $K > 0$ is the Landau-Ramanujan constant. We will discuss the function field analogue of Landaus theorem, and will report on recent developments on function field versions of some classical open problems in this theory.

Wednesday, July 22, 9:00, room A702.

Prime polynomial values of linear functions in short intervals

Efrat Bank (Tel Aviv University)

Abstract: In this talk I will present a function field analogue of a conjecture in number theory. This conjecture is a combination of several famous conjectures, including the Hardy-Littlewood prime tuple conjecture, conjectures on the number of primes in arithmetic progressions and in short intervals, and the Goldbach conjecture. I prove an asymptotic formula for the number of simultaneous prime values of n linear functions, in the limit of a large finite field. A key role is played by the computation of some Galois groups.

Wednesday, July 22, 10:30, room A702.

Analytic number theory in function fields

Zeev Rudnick (Tel Aviv University)

Abstract: I will discuss function field analogues of a number of open problems in analytic number theory, revolving around sums of arithmetic functions in short intervals and in arithmetic progressions. Examples include primes, the Mobius function and divisor functions. The solution of these problems rely on a mixture of techniques from analytic number theory, algebraic geometry and on equidistribution results.

Wednesday, July 22, 14:30, room A704.

A survey on some specialization theorems

Umberto Zannier (Scuola Normale Superiore di Pisa)

Abstract: We shall survey over the general issue of 'specializations which preserve a structure'. After recalling some typical examples and results, we shall discuss in a little more detail some specialization problems in the context of Unlikely Intersections and families of Pell's equations to be solved in polynomials; some proofs involve counting in certain Galois orbits.

Wednesday, July 22, 15:45, room A704.

Characterising $\mathbb{Q}$ by $G_{\mathbb{Q}} + \varepsilon$

Jochen Koenigsmann (University of Oxford)

Abstract: We show that if a field K has absolute Galois group $G_K := Gal(\widetilde{K}/K)$ isomorphic to $G_{\mathbb{Q}}$ then K shares many arithmetic properties with $\mathbb{Q}$, and in fact, very little extra information is needed in order to characterize $\mathbb{Q}$ up to isomorphism.

Thursday, July 23, 9:00, room A703.

Specialization of ℓ-adic Galois representations and Hilbertianity

Sebastian Petersen (Universität Kassel)

Abstract: Let S be a variety over $\mathbb{Q}$ with function field K and for every prime number ℓ let ρ_ℓ : $\mathrm{Gal}(K) \to \mathrm{GL}_n(\mathbb{Q}_\ell)$ be a continuous homomorphism which is unramified along S. For a closed point $s \in S$ with residue field $k(s)$ we can define the specialization $\rho_{\ell,s} : \mathrm{Gal}(k(s)) \to \mathrm{GL}_n(\mathbb{Q}_\ell)$ in the most natural way, and compare the image $Im(\rho_{\ell,s})$ of the specialization with the image $Im(\rho_\ell)$ of the generic representation. This leads to Hilbertianity questions, and there are interesting and recent results of Cadoret-Tamagawa, Ellenberg-Hall-Kowalski and others. These results can be applied when ρ_ℓ is the ℓ-adic representation of $\mathrm{Gal}(K)$ on the ℓ-adic Tate module of the generic fibre of a family of abelian varieties $\mathcal{A} \to S$ (or, more generally, on the ℓ-adic cohomology of the generic fibre of a morphism of schemes $X \to S$).

Thursday, July 23, 10:30, room A703.

On the number or ramified primes in specializations

François Legrand (Tel Aviv University)

Abstract: This talk will focus on the behavior of the number of ramified primes in finite Galois extensions of $\mathbb{Q}$ obtained by specializing finite Galois extensions of $\mathbb{Q}(T)$ at positive integers. In the first part, I will recall some previous results which consist in producing some *suitable* positive integers n such that the ramification (and then the number of ramified primes) of the specialization at n satisfies some desired properties and explain how these results relate to some strong versions of the Inverse Galois Problem. In the second part (based on a joint work with Lior Bary-Soroker), I will present new results about the behavior of the number of ramified primes at a *given* positive integer n.

Thursday, July 23, 14:30, room A702.

Arithmetic Statistics in Function Fields

Edva Roditty-Gershon (University of Bristol)

Abstract: I will discuss the mean square of sums of the generalised divisor function over arithmetic progressions for the rational function field over a finite field of q elements. In the limit as q tends to infinity we establish a relationship with a matrix integral over the unitary group, and analyse the integral. This is a joint work with Jon Keating, Brad Rodgers and Zeev Rudnick. I will also discuss the auto-correlations of arithmetic functions (in particular the generalised divisor function and the von Mangoldt function). Function field analogues of these problems have recently been resolved in the limit of large finite field size q. However, in this limit the correlations disappear: the arithmetic functions become uncorrelated. We compute averages of terms of lower order in q which detect correlations. Our results show that there is considerable cancellation in the averaging and have implications for the rate at which correlations disappear when q tends to infinity. This is a joint work with Jon Keating.