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Specialization of ℓ-adic Galois representations and Hilbertianity

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Abstract: Let S be a variety over $\mathbb{Q}$ with function field K and for every prime number ℓ let $\rho_\ell : \mathrm{Gal}(K) \to \mathrm{GL}_n(\mathbb{Q}_\ell)$ be a continuous homomorphism which is unramified along S. For a closed point $s \in S$ with residue field $k(s)$ we can define the specialization $\rho_{\ell,s} : \mathrm{Gal}(k(s)) \to \mathrm{GL}_n(\mathbb{Q}_\ell)$ in the most natural way, and compare the image $Im(\rho_{\ell,s})$ of the specialization with the image $Im(\rho_\ell)$ of the generic representation. This leads to Hilbertianity questions, and there are interesting and recent results of Cadoret-Tamagawa, Ellenberg-Hall-Kowalski and others. These results can be applied when ρ_ℓ is the ℓ-adic representation of $\mathrm{Gal}(K)$ on the ℓ-adic Tate module of the generic fibre of a family of abelian varieties $\mathcal{A} \to S$ (or, more generally, on the ℓ-adic cohomology of the generic fibre of a morphism of schemes $X \to S$).