Universität Konstanz
Department of Mathematics and Statistics
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Real Algebraic Geometry I – Exercise Sheet 4

Exercise 1 (4P). Show that there exists a Euclidean field that is not real closed.

Exercise 2 (4P). Let (R, P) an ordered field. Show that the following is equivalent:

(a) (R, P) is real closed.

(b) The intermediate value theorem for polynomials holds in (R, P): If $f \in R[X]$ and $a, b \in R$ such that $a \leq_P b$ and $\operatorname{sgn}_P(f(a)) \neq \operatorname{sgn}_P(f(b))$, then there is $c \in K$ with $a \leq_P c \leq_P b$ and $f(c) = 0$.

(c) If (L, Q) is an ordered extension field of (R, P) and $L|R$ is algebraic, then $(R, P) = (L, Q)$.

Exercise 3 (4P).

(a) Let R be a real closed field and K a subfield of R that is (relatively) algebraically closed in R. Show that K is also a real closed field.

(b) Show that there exists a countable real closed field.

Exercise 4 (4P). Let R be a real closed field and suppose that $f \in R[X]$ that has no roots in $R(\mathbb{i}) \setminus R$. Show that the derivative f' has again no roots in $R(\mathbb{i}) \setminus R$ unless it is zero.

Please submit until Thursday, November 24, 2016, 11:44 in the box named RAG I, Number 10, near to the room F411.