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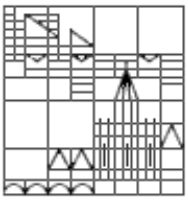

Konstanz, 12. April 2009

Real Algebraic Geometry and Emerging Applications

Research Proposal

Real algebraic geometry is concerned with specifically real questions in geometry and algebra. In the 19th century, classical algebraic geometers had a well-developed sense for the real point of view. While during the 20th century the general interest in this field had been somewhat uneven, it was only towards the end of the century that this began to change. Within the past two decades, the general perception of real algebraic geometry has changed in a way that could almost be called dramatic. Today, real algebraic geometry represents a very active and innovative area of research, rapidly developing at many ends, and with close ties to manifold other fields in- and outside of mathematics.

Hilbert (in the 1880s and 1890s) was the first to study the relation between positive polynomials and sums of squares. He considered this subject as so important that he included a central open question in his famous list of mathematical problems, the 17th problem. Its groundbreaking solution by Artin in 1927 opened the door for a systematic development of real algebra. However, it was not before the 1980s that real algebra and geometry started to be advanced thoroughly and with considerable emphasis. The discovery of new algebraic and geometric methods started to make problems accessible that formerly had been beyond hope.

From the 1990s on, the field gained additional momentum (still growing) through surprising interactions with other mathematical subjects, and also through the increasing general interest in applications. In the early 1990s, new results on positive polynomials started a flow of ideas between real algebraic geometry and the theory of moment problems from analysis, from which both fields have profited strongly to this date. The discovery of powerful new algorithms, combined with the enormously growing power of electronic computation in general, makes questions from real algebraic geometry a central issue in polynomial optimization since around the year 2000; conversely, the needs coming from problems in optimization shaped some of the recent directions that research in real algebraic geometry took. Relations to control theory, and to problems from theoretical physics and from engineering, lead to problems in noncommutative algebra related to notions of positivity. Largely, this area is still *terra incognita*, but analogues from the traditional commutative

theory (themselves only proved in the last 10 to 15 years) have already successfully served as excellent guiding principles, leading to startling recent results. For more details on all this, we refer to the project descriptions below.

This research proposal consists of three parts ("projects"):

1. *Positive polynomials, sums of squares and applications,*
2. *Linear matrix inequalities, semidefinite representations, hyperbolic polynomials and Lax-type questions,*
3. *Noncommutative real algebraic geometry and related problems.*

Below it will be demonstrated in detail that all three projects represent research areas that are currently developing very rapidly, and that are sharing substantial and widespread connections and applications to other areas of mathematics and mathematical physics. The scientific communities working on the questions addressed in these projects have traditionally been relatively unaware of each other. Only in the last few years, scientists from several sides have started to realize that these areas are unextricably linked with each other in many ways. The full extent of these connections is certainly not yet well understood.

It is the basic idea of this proposal to bring together expertise from the above areas in a more systematic way than has happened so far. We expect the intensified exchange of expert knowledge from these fields to bear fruit for all the involved research subjects, and to advance considerably the state of knowledge in all participating fields.

We believe that the scientists who are participating in this proposal are highly qualified for this program by their expertise, as demonstrated by their previous research records and monographs. We are also convinced that Konstanz is an excellent choice for carrying out this program. By its historic development, and with regard to its scientists, the *Schwerpunkt Reelle Geometrie und Algebra* of the University of Konstanz has already acquired an international reputation as a centre of real algebraic geometry for some years. Due to several replacements of professor positions — partially still pending, partially just completed — the Schwerpunkt is presently in the process of expanding significantly.