Donau–Rhein Modelltheorie und Anwendungen

Universität Konstanz

Talks

Talk 1 (11:30–12:15)

IP sets in ultraproducts of finite groups
(Daniel Palacín)

Abstract. A subset of an infinite group is called an IP set (for infinite-dimensional parallelepiped) if it contains all finite products of elements of an infinite sequence. Hindman's influential theorem states that for any finite coloring on the natural numbers, there is a monochromatic IP set. Outside the abelian context, Bergelson and Tao, taking over work of Gowers, have recently proved that a large subset of an ultraproduct of finite simple non-abelian groups is an IP set. In this talk I will explain how to prove this result by model-theoretic methods.

Talk 2 (13:30–14:15)
in conjunction with the Research Seminar "Real Algebraic Geometry"

Bounded motivic integral and motivic Milnor fiber
(Arthur Forey)

Abstract. Building on ideas of Hrushovski and Loeser, I will present a new motivic integration morphism, the bounded integral, that interpolates Hrushovski and Kazhdan's integrals with and without volume forms. It has applications to the complex and real motivic Milnor fibers. This is joint work with Yimu Yin.

Talk 3 (14:20–15:05)

Polynomial dynamical systems and Böttcher coordinates
(Harry Schmidt)

Abstract. In this talk I will discuss polynomial dynamical systems from an arithmetic viewpoint. In particular the use of Böttcher coordinates to attack diophantine problems connected to such systems.

Talk 4 (15:35–16:20)

On the Tree Structure of Orderings and Valuations

(Simon Müller)

Abstract. A quasi-ordered ring is a (possibly non-commutative) ring with 1 that is equipped with a binary, reflexive, transitive and total relation $\preceq$ such that $\preceq$ is in a certain sense compatible with $+$ and $\cdot$. It can be shown that any quasi-ordered ring is either an ordered ring or a valued ring and vice versa. Hence, quasi-orderings are a tool to study ordered and valued rings uniformly and simultaneously.

In this talk, given a unitary ring R, we introduce a coarser relation $\leq$ on the space $\mathcal{Q}(R)$ of all quasi-orderings on R. With the said relation, we subsume three different notions at once, namely coarsenings of valuations, compatibility of orderings and valuations, and inclusion of orderings. We then show that the quasi-orderings on R with fixed support form a tree, that way generalizing a result from Ido Efrat. As an application, we obtain that $(\mathcal{Q}(R), \leq)$ is a spectral set, i.e. the orderings and valuations on the (possibly non-commutative) ring R may be realized by the spectrum of some commutative ring.

Talk 5 (16:25–17:10)

Definable Valuations in Ordered Fields

(Lothar Sebastian Krapp)

Abstract. Let $\mathcal{L}_{\mathrm{r}} = \{+, -, \cdot, 0, 1\}$ be the language of rings and let $\mathcal{L}_{\mathrm{or}} = \mathcal{L}_{\mathrm{r}} \cup \{<\}$ be the language of ordered rings.

The study of definable valuations (i.e. valuations whose corresponding valuation ring is a definable set) in certain fields is motivated by the general analysis of definable subsets of fields as well as by recent conjectures on the classification of NIP fields. There is a vast collection of results giving conditions on $\mathcal{L}_{\mathrm{r}}$-definability of henselian valuations in a given field, many of which are from recent years. So far, not much seems to be known about $\mathcal{L}_{\mathrm{or}}$-definable valuations in ordered fields. Since $\mathcal{L}_{\mathrm{or}}$ is a richer language than $\mathcal{L}_{\mathrm{r}}$, it is natural to expect further definability results in the language of ordered rings.

In my talk, I will outline some progress in the study of $\mathcal{L}_{\mathrm{or}}$-definable valuations from my joint work with S. Kuhlmann and G. Lehéricy. In this regard, I will present sufficient topological conditions on the value group and the residue field of a henselian valuation v on an ordered field such that v is $\mathcal{L}_{\mathrm{or}}$-definable. Moreover, I will show how the study of $\mathcal{L}_{\mathrm{or}}$-definable valuations connects to ordered fields dense in their real closure as well as above mentioned conjectures regarding NIP fields.

All valuation theoretic notions will be introduced.