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PROSEMINAR ON LINEAR ALGEBRA – WS 2020/2021

List of possible topics

This is just a list of possible topics where you can find some ideas and references for preparing your seminars. Please, feel free to search for other sources and/or to propose to us a completely different topic in linear algebra on which you would like to build your own seminar.

1. **Operators on inner product spaces**
   (a) **Forms on inner product spaces and their representations**
   (See e.g. [4, Section 9.2, pp. 320–324].)
   (b) **Nonnegative and positive forms on inner product spaces**
   (See e.g. [4, Section 9.3, pp. 325–329].)
   (c) **Relation between positive operators, positive forms and positive matrices**
   (See e.g. [4, Section 9.3, pp. 329–331].)
   (d) **Forms and projections of vectors spaces**
   (See e.g. [4, Section 9.4, pp. 332–334].)
2. **Applications of the spectral theorem**
   (a) **Functions of diagonalizable normal operators**
   (See e.g. [4, Section 9.5, pp. 337–340].)
   (b) **Applications involving nonnegative operators**
   (See e.g. [4, Section 9.5, pp. 340–343].)
   (c) **Simultaneous diagonalization of families of normal operators**
   (See e.g. [4, Section 9.5, pp. 343–347].)
   (d) **Spectral theorem and primary decomposition**
   (See e.g. [4, Section 9.6, pp. 349–352].)
   (e) **Properties of normal operators on real vector spaces**
   (See e.g. [4, Section 9.6, pp. 352–354].)
   (f) **Unitarily equivalence of normal operators**
   (See e.g. [4, Section 9.6, pp. 354–358].)
3. **Bilinear forms**
   (a) **Bilinear forms on finite-dimensional vector spaces**
   (See e.g. [4, Section 10.1, pp. 360–366].)
   (b) **Symmetric bilinear forms and their diagonalization**
   (See e.g. [4, Section 10.2, pp. 367–372].)
   (c) **Skew-symmetric bilinear forms**
   (See e.g. [4, Section 10.3, pp. 375–379].)
   (d) **Group preserving bilinear forms**
   (See e.g. [4, Section 10.3, pp. 379–383].)

4. **Applications of Zorn's Lemma**

   (a) **Zorn's Lemma in the proof that any vector space has a basis**
   (See e.g. [1, Section 14.2].)

   (b) **Zorn's Lemma in the Hahn–Banach Theorem for linear functionals or in the proof of the existence of maximal ideals**
   (See e.g. [1, Section 14, pp. 235–247].)

   (c) **Tukey's Lemma: an equivalent version of Zorn's Lemma**
   (See e.g. [1, p. 245].)

5. **Applications of Jordan forms**

   (a) **Jordan form of a nilpotent matrix**
   (See e.g. [8, Section 4.2], [3, Section 6.5], [2, Section 57].)

   (b) **Applications of Jordan forms to systems of linear ordinary differential equations**
   (See e.g. [6, pp. 39–50].)

6. **Linear algebraic problems in mathematical physics**
   Topics connecting linear algebra and mathematical physics can be found for example in [7]. In particular, three mathematical problems appearing in quantum mechanical models are sketched in [7, Volume I, Chapter VIII, Section 11] and deal with:

   (a) **self-adjoint extensions of symmetric operators**
   (See e.g. [7, Volume II, Chapter X, Section 1].)

   (b) **spectral analysis**
   (See e.g. [7, Volume IV, Chapter XIII].)

   (c) **scattering theory**
   (See e.g. [7, Volume III, Chapter XI].)

7. **Further references**
   More interesting topics can be found in [2], [5], [3, Chapter VI].

References

[1] M. Carl, *Wie kommt man darauf?*, Springer Spektrum, Wiesbaden, 2017.

[2] P. R. Halmos, *Finite-dimensional vector spaces.* Reprinting of the 1958 second edition. Undergraduate Texts in Mathematics. Springer, New York, 1974.

[3] I. N. Herstein, *Topics in Algebra* (2nd ed.), John Wiley & Sons, 1975.

[4] K. Hoffman, R. Kunze, *Linear Algebra*, Prentice-Hall, Englewood Cliffs, N.J., 1971.

[5] F. Lorenz, *Lineare Algebra*, vol. I and II, Spektrum Akademischer Verlag, 2008.

[6] L. Perko, *Differential equations and dynamical systems* (3rd ed.), Texts in Applied Mathematics 7, Springer, New York, 2001.

[7] M. Reed, B. Simon, *Functional analysis* (2nd ed.), Methods of modern mathematical physics 1, Academic Press, San Diego, 1980.

[8] H. Shapiro, *Linear Algebra and Matrices: Topics for a Second Course*, American Mathematical Society, Providence, R.I., 2015.