



## Mathematik I

für die Studiengänge **Chemie, Life Science und Nanoscience**Freiwillige Zusatzaufgaben zur **Integralrechnung für reelle Funktionen**

### Lösungen

(1) Stammfunktionen sind

$$F_1(x) = -\frac{1}{5}(1-2x)^{\frac{5}{2}}$$

$$F_2(x) = x \ln(x) - x$$

$$F_3(x) = -\frac{1}{\pi} \cos(\pi x + 3)$$

$$F_4(x) = \frac{1}{5} \sin(5x)$$

$$F_5(x) = \frac{1}{5} \exp(1+5x)$$

$$F_6(x) = -\frac{1}{4(3+2x)^2}$$

$$(2) \int_1^2 \frac{x}{x^2+1} dx = \frac{1}{2}(\ln(5) - \ln(2)) \quad , \quad \int_0^2 \frac{x-1}{x+1} dx = 2 - 2\ln(3) \quad , \quad \int_0^1 \sqrt{2s+1} ds = \sqrt{3} - \frac{1}{3} \quad .$$

$$\int_0^1 \frac{6x^5+8x}{x^6+4x^2+1} dx = \ln(6), \quad \int_1^2 x^2 \ln(x) dx = \frac{8}{3} \ln(2) - \frac{7}{9}.$$

$$(3) \int \frac{2+3x}{4-9x^2} dx = -\frac{\ln(|2-3x|)}{3} + c, \quad \int \frac{1+\exp(4x)}{\exp(3x)} dx = -\frac{1}{3\exp(3x)} + \exp(x) + c,$$

$$\int x^2 \sin(x^3) dx = -\frac{\cos(x^3)}{3} + c, \quad \int \frac{\sqrt[3]{x^4} x^{\frac{1}{2}}}{x^{\frac{17}{6}}} dx = \ln(|x|) + c,$$

$$\int \frac{x^3+x}{x^4+2x^2+1} dx = \frac{1}{4} \ln(x^4+2x^2+1) + c$$

$$(4) \int_1^2 \frac{x}{\sqrt{x^2-1}} dx = \sqrt{3}, \quad \int_1^2 \frac{x}{x^2-1} dx = \infty \text{ (ex. nicht im eigentlichen Sinn)}, \quad \int_2^{\infty} \frac{1}{(x-1)^2} dx = 1$$

$$(5) (a) \int x^a \ln(x) dx = \frac{1}{a+1} x^{a+1} \left( \ln(x) - \frac{1}{a+1} \right) + c \quad \text{für } x > 0$$

$$(b) \int \frac{\ln(x)}{x} dx = \frac{1}{2} \ln(x)^2 + c \quad \text{für } x > 0.$$

$$(6) \quad f(x) = \frac{1}{x^2 - x - 2} = \frac{1}{3} \left( \frac{1}{x-2} - \frac{1}{x+1} \right) ,$$

$$\int_0^1 \frac{1}{x^2 - x - 2} dx = -\frac{2}{3} \ln(2) .$$

$$(7) \quad \int_1^2 \frac{1}{x^2 - 6x + 9} dx = \frac{1}{2} , \quad \int \frac{2x^2 - 14x + 18}{x^2 - 7x + 12} dx = 2x + 6 \ln \left( \left| \frac{x-3}{x-4} \right| \right) + c .$$