

Übungen zur **Mathematik I für Chemie, Life Science und Nanoscience**Freiwillige Zusatzaufgaben zur **Kombinatorik****Lösungen**

$$(1) \quad a_1 = \frac{3^2}{2} = \frac{9}{2} \quad a_2 = \frac{3^3}{2^2} = \frac{27}{4} \quad a_3 = \frac{3^4}{2^3} = \frac{81}{8} \quad a_i = \frac{3^{i+1}}{2^i}.$$

$$\sum_{i=0}^n a_i = 6 \left(\frac{3}{2} \right)^{n+1} - 6$$

$$(2) \quad \sum_{i=1}^7 \binom{9}{i} 2^3 = 4008, \quad \sum_{i=0}^9 \binom{10}{i} 5^{i+1} (-3)^{9-i} = \frac{5}{3} (5^{10} - 2^{10}) = 16274335,$$

$$\sum_{k=0}^{10} \binom{10}{k} \frac{1}{2^{2k}} = \left(\frac{5}{4} \right)^{10}, \quad \sum_{k=0}^{10} \frac{1}{2^{k+2}} = \frac{2047}{4096}.$$

$$(3) \quad \sum_{i=1}^{10} \prod_{l=1}^{10} 2 = 10240, \quad \prod_{k=2}^4 \sum_{j=0}^k (4-j) = 900, \quad \prod_{k=1}^4 \prod_{j=1}^k 2^{-j} = 2^{-20} = \frac{1}{1048576}.$$

$$(4) \quad \frac{N!}{N_1! N_2! N_3! N_4!}$$

$$(5) \quad \sum_{i=1}^{20} (2i+1) = 440, \quad \sum_{i=1}^n (2i+1) = (n+2)n, \quad \sum_{k=0}^{24} (\sqrt{k+1} - \sqrt{k}) = 5.$$

$$(6) \text{ a) } N = 9, \quad \text{b) } N = 4, \quad \text{c) } n = 30, \quad \text{d) } S_3 \left(\frac{1}{2}, -2 \right) = 2.$$

$$(7) \text{ a) } \frac{30!}{[6!]^5}, \quad \text{b) } \frac{30!}{1!8![7!]^3}, \quad \text{c) } \frac{30!}{2!4!6!8!10!}.$$

$$(8) \quad \frac{\sum_{i=0}^{10} \binom{10}{i} 3^{10-i}}{\sum_{k=0}^8 \binom{8}{k} 3^k (-1)^{8-k}} = 2^{12} = 4096$$
$$\sum_{k=1}^{25} \prod_{i=1}^k (i-1) = 0 \quad \text{da durch den Faktor zu } i=1 \text{ jeder Summand Null wird.}$$