Universität Konstanz
Fachbereich Mathematik und Statistik
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25th April 2011

Optimization
Exercises 2

✓ Exercise 5 (5 Points)

Consider the quadratic function $f : \mathbb{R}^n \to \mathbb{R}$,

$$f(x) = \frac{1}{2}x^\top Qx + c^\top x + \gamma \tag{1}$$

where $Q \in \mathcal{S}_n,\ c \in \mathbb{R}^n,\ \gamma \in \mathbb{R}$, with $\mathcal{S}_n$ the vector space of $n \times n$ symmetric matrices.

Show that Bemerkung 2.9 (cf. the scriptum of Prof. Volkwein) holds, i.e.,

(a) f is convex $\Leftrightarrow$ Q is positive semidefinite,
(b) f is strictly convex $\Leftrightarrow$ f is uniformly convex $\Leftrightarrow$ Q is positive definite.

Exercise 6

Consider the function in equation (1) with $Q \in \mathcal{S}_n$ symmetric and positive definite. Let $x^k \in \mathbb{R}^n$ arbitrary and $d^k \in \mathbb{R}^n$ be a descent direction of f in x^k.

Find the exact step-length t^k in direction d^k such that the decreasing of f is maximal.

Exercise 7

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a continuous function, $(x^k)_{k\in\mathbb{N}} \subseteq \mathbb{R}^n$ a sequence generated by the general descent method (Algorithmus 3.4).

Show that if x^* and x^{**} are two accumulation points of the sequence $(x^k)_{k\in\mathbb{N}}$, then $f(x^*) = f(x^{**})$ holds.

Exercise 8

Consider the general descent method (Algotithmus 3.4) for the function

$$f : \mathbb{R} \to \mathbb{R}, \qquad f(x) = x^2$$

with starting point $x^0 := 1$ and the direction d^k and step-size t^k:

(a) $d_k := -1$, $t_k := \left(\frac{1}{2}\right)^{k+2}$ with $k \in \mathbb{N}_0$,

(b) $d_k := (-1)^{k+1}$, $t_k := 1 + \frac{3}{2^{k+2}}$ with $k \in \mathbb{N}_0$.

Verify that these choices of the parameters for $k \in \mathbb{N}_0$ lead to a decreasing of the function f. In order to do that, present the sequence x^k generated by the Algorithmus 3.4 using induction with respect to k. Determine in each case $\lim_{k\to\infty} f(x^k)$ and compare them to the minimum of $f(x)$. Comment on the error!

Deadline: Monday, 2nd May, 8:30 am