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Numerik partieller Differentialgleichungen
1. Programm

Program 1 (FDM for the 2D Laplace equation) (8 Points)

Let $\Omega = (0,1)^2$. For each direction x and y consider $n \in \mathbb{N}$ inner discretization points with corresponding step size $h = \frac{1}{n+1}$, i.e., there is a total number of $n+2$ discretization points in each direction.

Solve numerically the Poisson problem

$$\begin{aligned} -\Delta u(x,y) &= f(x,y), \qquad (x,y) \in \Omega \\ u(x,y) &= g(x,y), \qquad (x,y) \in \partial\Omega \end{aligned} \tag{1}$$

with the classical finite difference method (i.e. five-point-stencil). Use the lexicographical ordering of the grid points in Ω.

Use the following functions f and g:

$$\begin{aligned} f(x,y) &= 4\pi \sin(2\pi x)(\pi \cos(2\pi y^2)(1+4y^2) + \sin(2\pi y^2)), \\ g(x,y) &= \sin(2\pi x)\cos(2\pi y^2); \end{aligned}$$

$$\begin{aligned} f(x,y) &= \begin{cases} 1, \text{ if } |x-0.5| + |y-0.5| \le 0.2, \\ 0, \text{ otherwise} \end{cases}, \\ g(x,y) &= 0; \end{aligned}$$

$$\begin{aligned} f(x,y) &= 0, \\ g(x,y) &= \begin{cases} 1, \text{ if } |x| \le 0.5, \\ 0, \text{ otherwise} \end{cases}. \end{aligned}$$

Show that in the first case, $u(x,y) = g(x,y)$, $(x,y) \in \Omega$ holds, and use this property to check your code for correctness.

Visualize the right hand side $f(x,y)$ and the numerical solution $u(x,y)$. Don't forget to lable the plots (`title, xlabel, ylabel, zlabel, ...`). Try different values for n. Document your code well and write a report including your observations.

Remark: For the Matlab implementation, the following commands can be useful: `ndgrid, mesh, spdiags, sparse`.