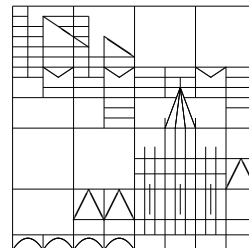


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 Wintersemester 2011/2012



Ausgabe: 2011/12/15
Abgabe: 2012/01/09

Numerik partieller Differentialgleichungen

1. Programm

Program 1 (FDM for the 2D Laplace equation) (8 Points)

Let $\Omega = (0, 1)^2$. For each direction x and y consider $n \in \mathbb{N}$ inner discretization points with corresponding step size $h = \frac{1}{n+1}$, i.e., there is a total number of $n + 2$ discretization points in each direction.

Solve numerically the Poisson problem

$$\begin{aligned} -\Delta u(x, y) &= f(x, y), & (x, y) \in \Omega \\ u(x, y) &= g(x, y), & (x, y) \in \partial\Omega \end{aligned} \quad (1)$$

with the classical finite difference method (i.e. five-point-stencil). Use the lexicographical ordering of the grid points in Ω .

Use the following functions f and g :

$$\begin{aligned} f(x, y) &= 4\pi \sin(2\pi x)(\pi \cos(2\pi y^2)(1 + 4y^2) + \sin(2\pi y^2)), \\ g(x, y) &= \sin(2\pi x) \cos(2\pi y^2); \end{aligned}$$

$$f(x, y) = \begin{cases} 1, & \text{if } |x - 0.5| + |y - 0.5| \leq 0.2, \\ 0, & \text{otherwise} \end{cases},$$

$$g(x, y) = 0;$$

$$f(x, y) = 0,$$

$$g(x, y) = \begin{cases} 1, & \text{if } |x| \leq 0.5, \\ 0, & \text{otherwise} \end{cases}.$$

Show that in the first case, $u(x, y) = g(x, y)$, $(x, y) \in \Omega$ holds, and use this property to check your code for correctness.

Visualize the right hand side $f(x, y)$ and the numerical solution $u(x, y)$. Don't forget to label the plots (`title`, `xlabel`, `ylabel`, `zlabel`, ...). Try different values for n . Document your code well and write a report including your observations.

Remark: For the MATLAB implementation, the following commands can be useful: `ndgrid`, `mesh`, `spdiags`, `sparse`.