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FB Mathematik & Statistik
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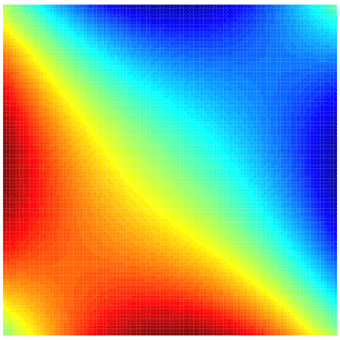
Parallele Numerik

Blatt 5

Problem 7: Heat equation



www.math.uni-konstanz.de/numerik/

Consider the following 2D heat equation

$$\begin{aligned} &\frac{\partial u}{\partial t} = \triangle u + f, \qquad (x, y) \in [0,1]^2, \quad t \geq 0, \\ &f = 2\pi^2 \sin(\pi x + \pi y). \end{aligned} \tag{1}$$

Please write a parallel program to numerically solve this equation by using the five-points finite difference scheme

$$\begin{aligned} \frac{u(t_{n+1}, x_i, y_j) - u(t_n, x_i, y_j)}{\triangle t} &= \frac{1}{h^2}\big[u(t_n, x_{i+1}, y_j) + u(t_n, x_{i-1}, y_j) \\ &+ u(t_n, x_i, y_{j+1}) + u(t_n, x_i, y_{j-1}) - 4u(t_n, x_i, y_j)\big] + f(t_n, x_i, y_j), \end{aligned} \tag{2}$$

$$\begin{aligned} &t_n = n\triangle t, \qquad n = 0, 1, 2, \ldots \\ &x_i = ih, \qquad i = 0, 1, \ldots, N, \\ &y_j = jh, \qquad j = 0, 1, \ldots, M. \end{aligned} \tag{3}$$

(i) Take u(0,:,:)=0 initially and boundary values as follows,

$$u(t,0,y) = -u(t,1,y) = \sin(\pi y),$$
$$u(t,x,0) = -u(t,x,1) = \sin(\pi x).$$

(ii) Choose $\triangle t \leq \frac{1}{4}h^2$ to ensure the stability of this scheme.

(iii) The analytic solution is stationary. Please choose a suitable convergence criterion ϵ. In case that

$$||u(n+1,:,:) - u(n,:,:)|| < \epsilon,$$

we say $u(n+1,:,:)$ is an approximate solution of equation (1).

(iv) Carry out a performance analysis of your parallel program.

(v) While using MPI,

- please compare performance with different communication modes (blocking/non-blocking);
- Remark: Write the parallel code in such a way that each node stores its final result in a separate file.

(vi) **A sequential program** is available online for reference.