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WS 1999/00

Tutorial *Dynamical Systems: Theory and Numerics*

Return: 12:00, Friday 28.01. metal box: Junk/Jungemann (48, ground floor)

Exercise 29:

Consider the nonlinear system

$$\dot{x} = Ax + H(x) \tag{1}$$

with $\|H(x)\| \leq c\|x\|^2, c \in \mathbb{R}$ and assume that the linear system $\dot{x} = Ax$ is asymptotically stable.

a) Show that the improper integral

$$B = \int_0^\infty \mathrm{e}^{tA^T} \mathrm{e}^{tA} \mathrm{d}t$$

is convergent.

b) Show that $V(x) = x^T Bx$ is a Ljapunov function for (1).

Exercise 30:

Consider the two–dimensional system

$$\dot{x} = -\frac{x}{2} - y^2$$
$$\dot{y} = -\frac{y}{4} + x^2$$

a) Check that the origin $(0, 0)$ is an equilibrium point and derive the linearization around $(0, 0)$.

b) Construct a strong Ljapunov function for the linear system and determine the stability of the origin.

c) Check that the Ljapunov function of b) is an appropriate Ljapunov function for the nonlinear system.

Exercise 31:

Consider the initial value problem

$$y'(x) = x - x^3, \qquad y(0) = 0 \tag{2}$$

1) Apply the explicit Euler method with step size h to derive an approximate solution of (2) for each $x_j = j \cdot h$.

2) Compute for each grid point the error between the exact solution and the numerical approximate and show that for x fixed and $h = x/n \to 0$ the error vanishes.