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WS 1999/00

Tutorial *Dynamical Systems: Theory and Numerics*

Return: 12:00, Friday 04.02. metal box: Junk/Jungemann (48, ground floor)

Exercise 32:

Implement the explicit Euler method for the general problem $\dot{x} = f(t,x)$, $x(0) = x_0 \in \mathbb{R}$. Apply it to the special case $f(t,x) = \lambda x$ and $x_0 = 1$ to calculate approximate solutions in the time interval $[0,1]$ with a time step Δt.

- Show that the size of Δt influences the accuracy of the solution and the computational time. Plot the grid errors and the approximate solutions together with the exact solution.
- Demonstrate the stiffness of the problem by choosing suitable examples (different λ and Δt).
- Show the first order convergence numerically by plotting the discretization error (which you calculate for different time steps) in logarithmic scale versus the time step. What do you expect and what do you find?

Always list the parameters with your plots and interpret your results.

Exercise 33:

Show that the one–step method defined by

$$\begin{aligned} k_1 &= f(t,x) \\ k_2 &= f(t+\frac{\tau}{2}, x+\frac{\tau}{2}k_1) \\ k_3 &= f(t+\tau, x+\tau(-k_1+2k_2)) \\ \psi &= \frac{1}{6}(k_1+4k_2+k_3) \end{aligned}$$

is of third order.

Exercise 34:

Implement the one–step method of Exercise 33 and compare the numerical results with the explicit Euler method and the exact solution for the initial value problem $\dot{x} = x$, $x(0) = 1$.