Prof. Dr. Helmut Neunzert
Dr. Michael Junk
Markus Jungemann

WS 1999/00

# Tutorial *Dynamical Systems: Theory and Numerics*

Return: 12:00, Friday 11.02. metal box: Junk/Jungemann (48, ground floor)

## Exercise 35:

Implement the implicit Euler method. Consider a problem $\dot{x} = \lambda x$, $x(0) = 1$ with $\lambda \ll 0$ and compare the results obtained with implicit and explicit Euler method.

## Exercise 36:

Show and explain that the second order Runge–Kutta method with incremental function given by

$$\psi(t, x, \tau) = f(t + \tau/2, x + \tau f(t, x)/2)$$

solves the differential equation $\dot{x} = -2at$ exactly.

## Exercise 37:

Implement "the" classical $4^{th}$ order Runge–Kutta method to integrate the initial value problem

$$x' = f(t, x), \quad x(t_0) = x_0 \in \mathbb{R}^d$$

Your program should be flexible concerning the following input parameters: dimension $d$, initial time $t_0$, final time $T$, initial condition $x_0$ and the number $n$ of integration steps with step size $\tau = (T - t_0)/n$. The right hand side $f$ should enter as a subroutine.

- Test your program for a simple initial value problem and document your numerical results carefully.
- Perform a numerical convergence analysis (discretization error versus step size) to determine the order of the method.
- Check the long term stability for the example $f(t, x) = \left(\begin{smallmatrix} x_2 \\ -x_1 \end{smallmatrix}\right)$. (Plot the phase curves!) Compare the result with those obtained from explicit and implicit Euler method.