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WS 1999/00

Tutorial *Dynamical Systems: Theory and Numerics*

Return: 12:00, Friday, 26.11. metal box: Junk/Jungemann (48, ground floor)

Exercise 11:

Consider the initial value problem $\dot{x} = t/(1+\sqrt{|x|})$ with $x(0) = 0$. Show that, although the assumptions of the Picard-Lindelöf Theorem are not satisfied (check Lipschitz continuity), the problem has a unique solution $x \in C^1((-\infty, \infty))$.

Exercise 12:

Let $\alpha > 0$, $n \in \mathbb{N}$ and $X_\alpha = C([0,\alpha], \mathbb{R}^n)$ the Banach space of $\mathbb{R}^n$ valued, continuous functions on $[0,\alpha]$ together with the usual maximum norm. Next, assume that $f : [0,\infty) \times \mathbb{R}^n \mapsto \mathbb{R}^n$ is continuous and define T as in Exercise 8.

a) Show that $T : X_\alpha \mapsto X_\alpha$ is continuous.

b) Show that T is compact (i.e. that the closure of $T(D)$ is compact in X_α for any bounded set $D \subset X_\alpha$).

c) Show that, for any $C > 0$, the set $D_{\alpha,C} = \{x \in X_\alpha : \|x(t)\| \leq C \quad \forall t \in [0,\alpha]\}$ is convex, closed and bounded in X_α.

c) Choose $\alpha, C > 0$ such that $T(D_{\alpha,C}) \subset D_{\alpha,C}$.

If (a)–(d) are satisfied, Schauder's fixed point theorem guarantees existence of $x \in D_{\alpha,C}$ such that $x = Tx$. Reformulate this relation as existence result for the initial value problem $\dot{x} = f(t,x)$, $x(0) = x_\circ$.

Exercise 13:

The simplest model for HIV dynamics is based on the equation $\dot{V} = P - cV$ where V is the HIV virus concentration in the blood (number/ml), P is an unknown function representing the rate of virus production, and c is a constant called the clearance rate constant. At $t = 0$ (time measured in days), a therapy is started which blocks the viral production and leads to the following evolution of the virus concentration

t	0	2	8	10	17	21	25
$V(t)$	49000	20500	9500	3750	1500	1300	600

Assuming that the therapy is perfect (i.e. causes $P = 0$ immediately), determine an approximate value for c from the data, the half–life of virus in the plasma (the time needed to reduce V by 50%), and the virus production P under the assumption that $V(t)$ was constant up to $t = 0$. How does the assumption of a perfect therapy influences the result? What happens if P drops to zero only with some delay?