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WS 1999/00

Tutorial *Dynamical Systems: Theory and Numerics*

Return: 12:00, Friday, 10.12. metal box: Junk/Jungemann (48, ground floor)

Exercise 17:

Consider two animal species of size $u(t)$ and $v(t)$ which follow the evolution

$$\dot{u} = 3u - u^2 - 2uv = g_1(u,v) \qquad \dot{v} = 4v - v^2 - 3uv = g_2(u,v)$$

- What do the separate terms in the equations tell you about the behavior within and between the species?
- Find the equilibrium points.
- Plot the lines $Z_i := \{(u,v) \,:\, g_i(u,v) = 0\}$ for $i = 1, 2$.
- The lines Z_1, Z_2 divide the first quadrant into subdomains $E_1, \ldots, E_4$. Find out whether orbits which start in E_i also remain there for $t \to \infty$ (Consider the direction field g). What does it mean for the species?

Exercise 18:

Consider a general Hamiltonian system $\dot{q} = \nabla_p H(q,p)$ and $\dot{p} = -\nabla_q H(q,p)$ where $q(t), p(t) \in \mathbb{R}^n$. Show that H is a first integral for the system.

Exercise 19:

Consider the mapping

$$T_t x = e^{-t} \begin{pmatrix} \cos t + \sin t & \sin t \\ -2\sin t & \cos t - \sin t \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \qquad x \in \mathbb{R}^2, t \in \mathbb{R}.$$

- Show that T_0 is the identity map on $\mathbb{R}^2$.
- Show that T_{-t} is the inverse of T_t for all $t \in \mathbb{R}$.
- Show that T_t is the phase flow related to a linear system $\dot{x} = A\dot{x}$ for some $A \in \mathbb{R}^{2\times 2}$.
- Plot the orbits corresponding to the initial values $x = (1,0)$ and $x = (0,1)$.
- For a sequence $t = 0, 0.2, 0.4, \ldots, 2$ plot the sets $T_t Q$ where Q is the unit square $Q = \{(x_1, x_2) \,:\, 0 \le x_1, x_2 \le 1\}$.