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WS 1999/00

Tutorial *Dynamical Systems: Theory and Numerics*

Return: 12:00, Friday, 17.12. metal box: Junk/Jungemann (48, ground floor)

Exercise 20:

Let $f(x) := \sum_{n=1}^{\infty} a_n x^n$ be an absolutely convergent series for $x \in \mathbb{R}$ with $|x| < r$ for some $r > 0$. Show that $f(A) := \sum_{n=1}^{\infty} a_n A^n$ is well defined for any matrix $A \in \mathbb{R}^{m\times m}$ with $\|A\| < r$ if $\|\cdot\|$ is a norm that satisfies $\|AB\| \leq \|A\|\,\|B\|$ for all matrices A, B.

- For which $A \in \mathbb{R}^{m\times m}$ is $\sin A$ and $\cos A$ well defined?
- Show that $(\exp(iA) + \exp(-iA))/2 = \cos(A)$ and $(\exp(iA) - \exp(-iA))/2i = \sin(A)$.
- Show that $(\cos A)^2 + (\sin A)^2 = I$, where I is the identity matrix.

Exercise 21:

Let $A \in \mathbb{R}^{2\times 2}$ and consider $\dot{x} = Ax$. Show that, by a suitable coordinate transformation $x = By$ with an invertible matrix $B \in \mathbb{R}^{2\times 2}$, the equation transforms into $\dot{y} = Cy$ with C of the form

$$a)\ \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \qquad b)\ \begin{pmatrix} \alpha & 0 \\ 1 & \alpha \end{pmatrix} \qquad c)\ \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}$$

and $\alpha, \beta \in \mathbb{R}$. How do the phase portraits for the cases (a), (b), (c) look like (consider all combinations $\alpha\beta > 0, \alpha\beta = 0, \alpha\beta < 0$).

Exercise 22:

Assume a radioactive material S_1 is given which decays with rate λ_1 into a material S_2 which again decays with rate λ_2 into a stable material S_3.

- Derive a system of ODEs for the masses $m_i(t)$ of the materials S_i (specify the system matrix).
- Calculate the fundamental matrix of the system.
- Solve the initial value problem with $m_1(0) = M$ and $m_2(0) = m_3(0) = 0$.