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WS 1999/00

Tutorial *Dynamical Systems: Theory and Numerics*

Return: 12:00, **Thursday, 23.12. !!** metal box: Junk/Jungemann (48, ground floor)

Exercise 23:

Consider the 3×3 matrix

$$A = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

- Find a basis of principal vectors of A.
- Represent A in this basis.
- Calculate $\exp(A)$ and $\ln(A)$.

Exercise 24:

Many practically relevant processes can be described by *compartment models* (compare, for example, Exercise 22): Let $K_1, \ldots, K_n$ be n real or fictitious compartments. Assume K_i contains a substance of mass $m_i(t)$. Assume further that there is a transport mechanism from compartment K_i to $K_j \neq K_i$ with rate $k_{ij} \geq 0$.

a) Derive a system of ODEs for $m(t) = (m_1(t), \ldots, m_n(t))^T$.

b) Show that the total mass $\sum_{i=1}^n m_i(t)$ is constant.

c) Show that $m_i(0) \geq 0$ for all i implies $m_i(t) \geq 0$ for all i and all $t \geq 0$.

d) How does the assumption of external mass sources $s_i(t) \geq 0$ for the compartments K_i changes the results of (a), (b) and (c)?

Exercise 25:

Consider the differential equation $\ddot{\theta} + b(t)\theta = 0$ with a p–periodic, continuous function $b : \mathbb{R} \mapsto \mathbb{R}$. Let $Y(t)$ be a fundamental matrix of the corresponding first order system. Show that $\det Y(t) = 1$. Using Floquet theorem, show that the product of the characteristic roots is equal to one. What are the consequences?