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WS 1999/00

Tutorial *Dynamical Systems: Theory and Numerics*

Return: 12:00, Friday 21.01. metal box: Junk/Jungemann (48, ground floor)

Exercise 26:

Consider the linear system $\dot{x} = A(t)x$ with $A : \mathbb{R} \mapsto \mathbb{R}^{n\times n}$ continuous.

- Show that the solution $x = 0$ is attractive if and only if it is asymptotically stable.
- Let $\lambda_{\max}(t)$ be the maximal eigenvalue of $A(t) + A^T(t)$ where A^T is the transpose of A. Show that if

$$\lim_{t\to\infty} \int_{t_0}^{t} \lambda_{\max}(s)\, ds = -\infty \qquad \forall t_0 \in \mathbb{R}$$

  then the zero solution is asymptotically stable.
  (Hint: Set up a differential equation for $\|x(t)\|^2$.)

Exercise 27:

Consider the systems

$$\dot{x} = y + x^3 \qquad \dot{y} = -x + y^3 \tag{1}$$
$$\dot{x} = y - x^3 \qquad \dot{y} = -x - y^3 \tag{2}$$

Show that $(x, y) = (0, 0)$ is a stationary solution which is linearly stable in both cases but which is unstable in case (1) and asymptotically stable in case (2).

Exercise 28:

In a long–term drug therapy, a certain amount of the drug is taken regularly. The medical substance is transported in the blood system, it can enter and leave the tissue and is excreted at a certain rate.

- Set up a dynamical system to describe the drug level in blood and tissue.
- Can you tell something about the drug level after a long time period?