



Übungen zu Numerik PDGL II

Blatt 01

Aufgabe 1:

We consider the following functional

$$J(u) = \frac{1}{2} \int_0^1 |u'(x)|^2 dx - \int_0^1 f(x)u(x)dx, \quad u \in H^1(0,1). \quad (1)$$

1) Prove that, at a local minimum u of J , the following condition holds

$$\int_0^1 u'(x)v'(x)dx - \int_0^1 f(x)v(x)dx = 0, \quad v \in H^1(0,1). \quad (2)$$

Note: show that all directional derivatives of J disappear at u , i.e. $\frac{d}{d\epsilon}J(u+\epsilon v)|_{\epsilon=0} = 0$.

2) Show that f having zero integral average is a necessary condition for the existence of a minimizer of (1).

3) Show that minimizers of $J(u)$ are not unique.

4) Derive the PDE (strong form) belonging to the weak formulation (2) by choosing appropriate test functions and setting suitable assumptions for u and f .

Aufgabe 2:

1) Implement the class **vector** completely by overloading relevant operators (from matlab: docsearch overloading), such as **uplus**, **plus**, **uminus**, **minus**, **mtimes**, **eq**, **ne**, **display**.

2) Similar as 1), implement the classes **linearForm** and **bilinearForm** including the related apply methods.

3) For **bilinearForm** also implement **mldivide**(backslash) to return the **vector** u which satisfies the variational equation related to the forms.

Finally test your implementations with appropriate examples.