Dr. Michael Junk
Dipl. Math. techn. Dirk Kehrwald

SS 2000

Tutorial *Analytical Methods for PDEs*

Return: Monday, 26.06. 1pm, metal box (Kehrwald), ground floor bldg. 48

Exercise 26:

Let $\Omega \subset \mathbb{R}^2$ be the unit square and let $g(x,y) = (1-|x-4/7|)(1-|y-3/8|)$. Use separation of variables and any math–package (maple, matlab, etc.) to find an approximate solution of $\Delta u = 0$ in Ω and $u = g$ on $\partial\Omega$.

a) Compare the Fourier–sine approximations with the exact boundary values (after truncating the Fourier expansion at $M = 5$ and $M = 10$) at the northern and eastern boundary (plots!).

b) Give an isoline plot of the solution for $M = 10$.

c) Give an estimate for the maximal error of the approximation ($M = 10$) in Ω.

Exercise 27:

In the half plane $\{(t,x) : t > 0,\ x \in \mathbb{R}\}$, we consider

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial x^2} = 0 \qquad u(0,x) = \phi(x), \quad \frac{\partial u}{\partial t} = 0.$$

Show that this initial value problem for Laplace's equation is not well posed by considering initial values

$$\phi_n(x) = e^{-\sqrt{n}} \sin(nx) \qquad n \in \mathbb{N}$$

and following the steps:

- Find a solution of the form $u_n(t,x) = f_n(t) g_n(x)$.
- Show that ϕ_n converges to zero for $n \to \infty$, uniformly in x.
- Show that $sup_{x \in I} |u_n(t_0, x)|$ diverges for every interval $I \subset \mathbb{R}$ and every $t_0 > 0$.

This shows, that the solution does not depend continuously on the data in any reasonable topology which is stronger than local uniform convergence on $\mathbb{R}^2$.

Exercise 28:

Show that there is no general maximum principle for solutions of the problem

$$\Delta\Delta u(\boldsymbol{x}) = 0, \qquad \boldsymbol{x} \in \Omega$$

where Ω is a domain in $\mathbb{R}^d$.