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Tutorial *Analytical Methods for PDEs*

Return: Monday, 03.07. 1pm, metal box (Kehrwald), ground floor bldg. 48

Exercise 29:

Consider the functions

$$E_1(\boldsymbol{x}) = -\frac{\cos|\boldsymbol{x}|}{4\pi|\boldsymbol{x}|}, \qquad E_2(\boldsymbol{x}) = \frac{\sin|\boldsymbol{x}|}{4\pi|\boldsymbol{x}|}, \qquad \boldsymbol{x} \in \mathbb{R}^3.$$

Prove that E_1 is a fundamental solutions of the operator $L = \Delta + 1$ on $\mathbb{R}^3$. Show that E_2 satisfies $LE_2 = 0$ on $\mathbb{R}^3$ (i.e. there is *no* Liouville theorem for L because E_2 is a bounded solution of $LE_2 = 0$ which is *not* constant). Use E_2 to show that there is no maximum principle for L?. (General hint: you can use parts of Exercise 24.)

Exercise 30:

Let $\Omega = \{\boldsymbol{x} \in \mathbb{R}^d : |\boldsymbol{x}| > 1\}$ and let $f : \Omega \mapsto \mathbb{R}$ and $g : \partial\Omega \mapsto \mathbb{R}$ be continuous. Show that solutions of the problem

$$\Delta u = f \quad \text{in } \Omega, \qquad u = g \quad \text{on } \partial\Omega$$

are unique under the additional condition

$$\sup_{|\boldsymbol{x}|>R} |u(\boldsymbol{x})| \xrightarrow[R\to\infty]{} 0$$

Exercise 31:

Construct a Green's function for the Dirichlet Poisson problem

$$\Delta u = f \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega$$

with the help of the method of images if Ω is the quarter plane

$$\Omega = \{x \in \mathbb{R}^2 \;:\; x_1, x_2 > 0\}.$$

Exercise 32:

Construct a Green's function for the Neumann Poisson problem

$$\Delta u = f \quad \text{in } \Omega, \qquad \frac{\partial u}{\partial n} = 0 \quad \text{on } \partial\Omega$$

with the help of the method of images if Ω is the half space

$$\Omega = \{x \in \mathbb{R}^2 \;:\; x_1 > 0\}.$$