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Tutorial *Analytical Methods for PDEs*

Return: Monday, 08.05. 1pm, metal box (Kehrwald), ground floor bldg. 48

Exercise 7:

Show that the Cauchy problem for the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad u(0,x) = u_0(x),\ \frac{\partial u}{\partial t}(0,x) = v_0(x),\ x \in \mathbb{R}$$

is well posed in $C_b^k([0,T] \times \mathbb{R})$ with $k \geq 2$ and data $u_0 \in C_b^k(\mathbb{R})$ and $v_0 \in C_b^{k-1}(\mathbb{R})$.

Exercise 8:

Show that the spherical average of a C^2-function f

$$I(\boldsymbol{x}, r) = \frac{1}{4\pi} \int_{\partial B_1(0)} f(\boldsymbol{x} + r\boldsymbol{y})\, dS(\boldsymbol{y}), \qquad \boldsymbol{x} \in \mathbb{R}^3,\ r > 0$$

satisfies the Euler-Poisson-Darboux equation

$$r \Delta_{\boldsymbol{x}} I(\boldsymbol{x}, r) = \frac{\partial^2}{\partial r^2}(r I(\boldsymbol{x}, r)).$$

(Hint: use $\int_0^r s^2 I(\boldsymbol{x}, s)\, ds$ and the divergence theorem.)

Exercise 9:

We consider a continuum of particles of mass $m > 0$ which move in some force field $\boldsymbol{F} : \mathbb{R}^3 \mapsto \mathbb{R}^3$, i.e. they follow trajectories given by $\dot{\boldsymbol{x}} = \boldsymbol{v}$ and $m\dot{\boldsymbol{v}} = \boldsymbol{F}(\boldsymbol{x})$. Let $f(t, \boldsymbol{x}, \boldsymbol{v})$ denote the mass density of particles at position $\boldsymbol{x} \in \mathbb{R}^3$ with velocity $\boldsymbol{v} \in \mathbb{R}^3$ at time $t \in \mathbb{R}$ so that

$$\int_A \int_B f(t, \boldsymbol{x}, \boldsymbol{v})\, d\boldsymbol{v}\, d\boldsymbol{x}, \qquad A, B \subset \mathbb{R}^3$$

is the total mass of particles in the volume A having velocities in B. Use the transport theorem to derive a partial differential equation for the time evolution of f under the assumption that particles are neither destoyed nor created.