Dr. M. Junk WS 2001/02-10

Tutorial *Analysis I*

Return: 12:00, Friday 25.01.2002

Exercise 39:

Determine the precise interval of convergence of the following power series:

$$\sum_{n=0}^{\infty} \sqrt{n} x^n, \qquad \sum_{n=0}^{\infty} x^{n!}, \qquad \sum_{n=0}^{\infty} 3^n x^{2n+1+(-1)^n}$$

Moreover, construct a power series with the interval of convergence $(-2, 2]$.

Exercise 40:

For $n = 0, 1, 2, 3, \ldots$ let $a_n = (\frac{4+2(-1)^n}{5})^n$.

(i) Determine $\limsup(a_n)^{1/n}$, $\liminf(a_n)^{1/n}$, $\limsup |a_{n+1}/a_n|$, $\liminf |a_{n+1}/a_n|$.

(ii) Do the series $\sum a_n$ and $\sum(-1)^n a_n$ converge? Justify.

(iii) Determine the interval of convergence of the power series $\sum a_n x^n$.

Exercise 41:

Use the definitions

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}, \qquad \sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1},$$

to prove the relation $\sin(2x) = \sin(x)\cos(x)/2$.

Exercise 42:

Let $f_n(x) = \frac{nx}{1+n^2x^2}$ for $x \in \mathbb{R}$. Sketch a few members f_n of this sequence and prove your conjectures about the following questions:

(i) What is the *pointwise limit* f of the sequence (f_n)?

(ii) Does the sequence converge *uniformly* on $[0, 1]$?

(iii) Does the sequence converge *uniformly* on $[1, \infty)$?

Apply the same considerations to $f_n(x) = (x - 1/n)^2$, $x \in \mathbb{R}$.