Dr. M. Junk WS 2001/02-13

Tutorial *Analysis I*

Return: 12:00, Friday 15.02.2002

Exercise 50:

Let f be differentiable on $\mathbb{R}$, $f(0) = 0$ and $1 \leq f'(x) \leq 2$ for $x \in \mathbb{R}$. Show that $|x| \leq |f(x)| \leq 2|x|$ for all $x \in \mathbb{R}$.

Exercise 51:

Determine the following limits:

$$\lim_{x \to 0} \frac{\tan x - x}{x^3}, \qquad \lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right).$$

Exercise 52:

Let f, g be differentiable on $\mathbb{R}$ with $f' = g$, $g' = f$, and $f(0) = 1, g(0) = 0$. Derive the Taylor series of f and g and determine their radius of convergence. Show that the Taylor series converge to f resp. g. Establish a connection between the power series of $f + g$ and $f - g$ and the exponential series and express f, g explicitly in terms of the exponential function.

Exercise 53:

Let $f(x) = x$ for rational x and $f(x) = 0$ for irrational x. Calculate the upper and lower Darboux integral of f on the interval $[0, b]$. Is f integrable on $[0, b]$?