Dr. M. Junk WS 2001/02-1

Tutorial *Analysis I*

Return: 12:00, Friday 09.11., box: Venkitesvaran

Remark: You can return your exercises either alone, or in a group with up to three students. The members of a group should not change during the semester. The homework is returned at the metal boxes next to lecture hall 48/210. Information can also be found on the homepage:
`http://www.mathematik.uni-kl.de/~junk/analysis/analysis1.html`

Exercise 1:

Using the field axioms, show that for $x, y \in \mathbb{R}$

(i) $-0 = 0$,

(ii) $-(-x) = x$

(iii) $0x = 0$

(iv) $(-1)x = -x$

(v) $(-x)(-y) = xy$

(vi) $(-x)y = -(xy)$

(vii) $xy = 0 \iff x = 0$ or $y = 0$,

(viii) if $a \neq 0$ then $ax = y$ has the unique solution $x = a^{-1}y$,

(ix) $(x^{-1})^{-1} = x$ für $x \neq 0$.

Indicate the axiom (or the number of a previously shown result) in every step of your proofs.

Exercise 2:

Show (as in Exercise 1) with field and order axioms for $x, y, z \in \mathbb{R}$

(i) $x \leq y \Longrightarrow -y \leq -x$,

(ii) $x \leq y$, $z \leq 0 \Longrightarrow yz \leq xz$,

(iii) $0 \leq x$, $0 \leq y \Longrightarrow 0 \leq xy$,

(iv) $0 \leq xx = x^2$,

(v) $0 < 1$,

(vi) $0 < x \Longrightarrow 0 < x^{-1}$,

(vii) $0 < x < y \Longrightarrow 0 < y^{-1} < x^{-1}$.

Exercise 3:

Guess a formula for the direct calculation of $1 + 3 + 5 + \cdots + (2n - 1)$ for every $n \in \mathbb{N}$ and prove the formula by induction.

Exercise 4:

Show that, for every $b \geq -1$ and $n \in \mathbb{N}$, we have $(1 + b)^n \geq 1 + nb$. (Rem.: x^n is an abbreviation for the n-fold product $x \cdots x$.)

Exercise 5:

Show that $11^n - 4^n$ is divisible by 7 (without remainder) when $n \in \mathbb{N}$.