Tutorial *Analysis I*

Return: 12:00, Friday 23.11.

Exercise 11:

For each of the sequences

$$\frac{n^3+100n}{100n^2+n}, \qquad n!, \qquad \frac{\sin(n)}{n}, \qquad \frac{1+2+\cdots+n}{n^2},$$
$$\frac{n^3-12n^2+44n-48}{2n^3-12n^2+22n-10}, \qquad \frac{3^n}{n!}, \qquad \sin(n), \qquad \sqrt{4n^2+n}-2n.$$

(i) Sketch the behavior of the sequence (it is advisable (!) to use a computer; choose an index range, for instance $n = 1, 2, \ldots, 50$, plot the points (n, s_n), and return a printout of the graphic).

(ii) "Guess" whether the sequence converges or diverges and, in the case of convergence, specify the limit.

(iii) Prove the conjecture in (ii) by using the definition of convergence and suitable estimates (not the limit theorems).

(You can use your school-knowledge about the $\sin(\cdot)$ and $\sqrt{\cdot}$ functions.)

Exercise 12:

Let (s_n) be a sequence in $\mathbb{R}$. Show: if $\lim s_n = s$ exists then $\lim |s_n| = |s|$. Under which condition is the converse statement also true?

Exercise 13:

Let (s_n) be a convergent sequence with $\lim s_n > s$. Show that there exists a number N such that $s_n > s$ for all $n > N$.

Exercise 14:

Prove the limit theorems for convergent sequences (x_n), (y_n):

$$\lim(x_n + y_n) = \lim x_n + \lim y_n, \qquad \lim(x_n y_n) = \lim x_n \lim y_n$$

and, if $y_n \neq 0$ for all $n \in \mathbb{N}$, $\lim y_n \neq 0$, then $\lim(x_n/y_n) = \lim x_n / \lim y_n$.