Dr. M. Junk WS 2001/02-5

Tutorial *Analysis I*

Return: 12:00, Friday 07.12.

Exercise 18:

Starting from a sequence $(s_n)_{n\in\mathbb{N}}$, we define $t_n = (s_1 + s_2 + \cdots + s_n)/n$. Show:

(i) If (s_n) converges then (t_n) converges also. What is the limit of (t_n) in that case?

(ii) If (s_n) is periodic (i.e. there exists $p \in \mathbb{N}$ such that $s_{n+p} = s_n$ for all $n \in \mathbb{N}$), then (t_n) converges.

(iii) It can happen that (s_n) diverges but (t_n) converges.

(iv) It can happen that (s_n) and (t_n) are bounded and both diverge.

Exercise 19:

A non-empty set D is called countable if there exists a surjective mapping $\mathbb{N} \to D$, i.e. if there is a sequence $(x_n)_{n\in\mathbb{N}}$ with $D = \{x_n : n \in \mathbb{N}\}$ (and $D \neq \emptyset$ is called uncountable if D is not countable). Show the countability of the following sets by specifying an appropriate sequence:

$$D = \{a_1, \ldots, a_n\}, \qquad D = \mathbb{N}, \qquad D = \mathbb{Z}.$$

In addition, show that if $\mathcal{D} = \{D_i : i \in \mathbb{N}\}$ is a countable set of countable sets D_i, then the union of all sets D_i is again countable. (Hint: recall how $\mathbb{Q}$ can be written as sequence and use a similar trick.)

Exercise 20:

Let $(s_n)_{n\in\mathbb{N}}$ be a sequence with $s_n \neq 0$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} \left|\frac{s_{n+1}}{s_n}\right| = L \in \mathbb{R}$. Prove that $\lim_{n\to\infty} |s_n| = \infty$ if $L > 1$.