Dr. M. Junk WS 2001/02-8

Tutorial *Analysis I*

Return: 12:00, Friday 11.01.2002

Exercise 30:

Show by direct application of the $\epsilon - \delta$ definition that the following functions are continuous on $\mathbb{R}$:

$$f(x) = \sqrt{|x|}, \qquad f(x) = \frac{1}{x^4+1}.$$

Exercise 31:

Let f, g be continuous functions with domain $(a,b) \subset \mathbb{R}$. Show that $f = g$ if f and g coincide in all rational points $r \in \mathbb{Q} \cap (a,b)$. Show also that the function

$$h(x) = \begin{cases} 0 & x \in [0,1] \backslash \mathbb{Q} \\ x & x \in [0,1] \cap \mathbb{Q} \end{cases}$$

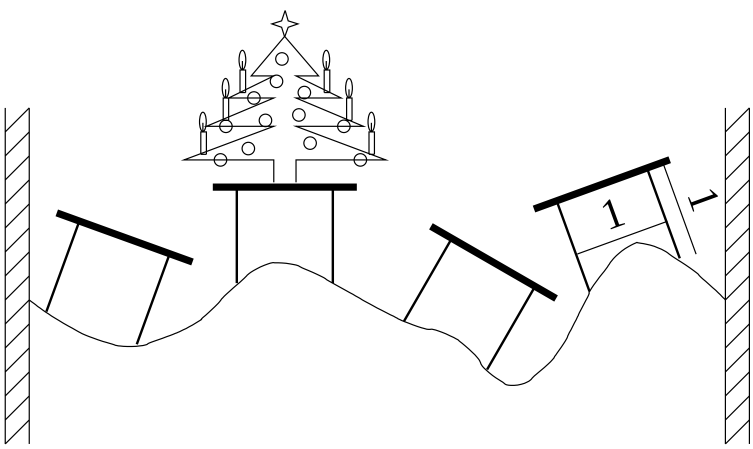
is continuous in $x = 0$ and that this is the only point of continuity. Define a function on $[0,1]$ which is discontinuous in every point.

Exercise 32:

Prove that a polynomial $P(x) = a_0 + a_1 x + \cdots + a_n x^n$, $a_n \neq 0$, $x \in \mathbb{R}$ with odd degree n has at least one root.

Exercise 33:

A (admittedly unrealistic, two-dimensional) table should be set up in a room with uneven floor in such a way that the table top is exactly horizontal. The length and distance of the legs are supposed to be $L = 1$ and the room has width $B = 5$. Is there always a position in the room at which a horizontal placement is possible? Or are there certain minimal requirements on the shape of the floor?



Merry Christmas
and a Happy New Year