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Tutorial *Integration of high-dimensional problems*

Exercise 1:

Integrate the function

$$f(\boldsymbol{x}) = |\boldsymbol{x}|^2 = \sum_{i=1}^{d} x_i^2, \qquad \boldsymbol{x} \in I^d$$

with the Hit-or-Miss Monte Carlo method for several dimensions $d \in \mathbb{N}$.

- use a fixed number of points
- plot the relative error versus the dimension
- what kind of dependence on the dimension do you expect?

Exercise 2:

Apply the Hit-or-Miss Monte Carlo method to

$$f(x) = \sin(x), \qquad x \in [0, \pi].$$

Plot the random points and the function graph in a 2d diagram.

Exercise 3:

Apply the Sample-Mean Monte Carlo method to the function

$$f(x) = (1+\beta)x^{\beta}, \qquad x \in [0,1]$$

for various values of $\beta \in (-1, 0)$.

- plot the error versus $\log_{10} N$ (N is the number of random points used in the calculations)
- find the rate of convergence using regression
- calculate the integral with the deterministic approximation

  $$\int_0^1 f(x)\,dx \approx \frac{1}{N}\sum_{i=1}^{N} f\left(\frac{i-1/2}{N}\right)$$

  and perform the same convergence analysis as for MC integration.
- what happens for $\beta < -1/2$?

Exercise 4:

Repeat the analysis in Exercise 3 for the case

$$f(x) = \sin(1/x) \qquad \text{with} \qquad \int_0^1 \sin(1/x)\, dx \approx 0.5040670619, \tag{1}$$

and

$$f(x) = x^{-1/2}\sin(20/x) \qquad \text{with} \qquad \int_0^1 x^{-1/2}\sin(20/x)\, dx \approx 0.02357257. \tag{2}$$

(By the way, how would you find the given integral values?)

Exercise 5:

Use the Sample-Mean Monte Carlo method to integrate the function

$$f(\boldsymbol{x}) = \Pi_{i=1}^{d}(1+\beta)x_i^{\beta}, \qquad \boldsymbol{x} \in I^d$$

with $\beta = -1/8$.

- use a fixed number of points
- plot the relative error versus the dimension
- what kind of dependence on the dimension do you expect?

Exercise 6:

- Test the built-in random number generator with the chi-square goodness-of-fit test and the Kolmogorov-Smirnov statistic.
- Construct a mixed congruential generator and test its efficiency with the same tests.
- After finding a good multiplier, increment, and modulus apply your generator to any of the integration problems above.
- What happens if you take a generator of low quality?