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Tutorial *Integration of high-dimensional problems*

Exercise 7:

- Implement a generator for the van der Corput sequence in base b. If you use matlab, check whether your generator is faster than the one given on the lecture homepage (you find it under "Matlab source code" at the bottom of the page).
- Use the van der Corput generator to construct Halton sequences for a given list of bases.
- Choose a pair of bases and plot the corresponding firstHalton points in a 2D diagram (e.g. 1000 points). Repeat for other choices of the bases.

Exercise 8:

Integrate the function

$$f(\boldsymbol{x}) = |\boldsymbol{x}|^2 = \sum_{i=1}^{d} x_i^2, \qquad \boldsymbol{x} \in I^d$$

with Halton points in several dimensions $d \in \mathbb{N}$.

- Use a fixed number of points and
- plot the relative error versus the dimension.
- Compare with Sample-Mean Monte Carlo.
- Calculate the variation of f (using the integral formula).

Exercise 9:

In dimension $d = 10$, integrate the functions

$$f(\boldsymbol{x}) = |\boldsymbol{x}|^2 = \sum_{i=1}^{d} x_i^2, \qquad f(\boldsymbol{x}) = \prod_{i=1}^{d} \sin(\pi x_i), \qquad \boldsymbol{x} \in I^d$$

with Halton points.

- Plot the relative error versus $\log_{10} N$ (N is the number of points used in the calculations).
- Find the rate of convergence using regression.
- Calculate the integral with Sample-Mean Monte Carlo and perform the same numerical convergence analysis.

Exercise 10:

Integrate the function

$$f(\boldsymbol{x}) = \exp\left(\sin(\pi|\boldsymbol{x}|^2)\right), \qquad \boldsymbol{x} \in I^d$$

for $d = 2, 3, \dots, 10$. (Maple gives 1.680102051 in 2D - and breaks down for 3D)

Exercise 11:

Integrate the function

$$f(\boldsymbol{x}) = \begin{cases} 1 & x_1 < x_2 \\ 0 & x_1 \geq x_2 \end{cases}, \qquad \boldsymbol{x} \in I^2$$

and proceed as in Exercise 9. Are you surprised by the result?

Exercise 12:

For some $\alpha \in (0, 1)$, consider the functions

$$f_\alpha(\boldsymbol{x}) = \begin{cases} 1 & x_i < \alpha \quad \forall i \\ 0 & \text{else} \end{cases}, \qquad F_\alpha(\boldsymbol{x}) = \begin{cases} 1 & x_i > 1 - \alpha \quad \forall i \\ 0 & \text{else} \end{cases}, \qquad \boldsymbol{x} \in I^d$$

Calculate the exact values of the integrals and use Halton points to determine approximate values.

- Plot the relative error for fixed N, d and varying $a \in (0, 1)$.
- Plot the relative error for fixed N, a and varying $d \in \mathbb{N}$.
- Does the variation in the sense of Hardy and Krause play a role?

Exercise 13:

Implement the Sobol and Faure generators. You can also use the code from the lecture's homepage (if possible, increase the speed of these programs). Then, repeat the exercises by using and comparing different generators.