Universität Konstanz
Fachbereich Mathematik und Statistik
Prof. Dr. Michael Junk
M. Gubisch, O. Lass, R. Mancini, S. Trenz
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Optimierung
2. Übungsblatt

☐ Exercise 5

Consider the quadratic function $f : \mathbb{R}^n \to \mathbb{R}$,

$$f(x) = \frac{1}{2}\langle x, Qx\rangle + \langle c, x\rangle + \gamma$$

where $c \in \mathbb{R}^n$, $\gamma \in \mathbb{R}$ and $Q \in \mathbb{R}^{n\times n}$ is a symmetric matrix.

Show directly (without using Satz 2.10) that

$$f \text{ is convex} \Leftrightarrow Q \text{ is positive semidefinite.}$$

☐ Exercise 6

Consider the function

$$f(x) = \frac{1}{2}\langle x, Qx\rangle + \langle c, x\rangle + \gamma$$

with $Q \in \mathbb{R}^{n\times n}$ symmetric and positive definite. Let $x^k \in \mathbb{R}^n$ arbitrary and $d^k \in \mathbb{R}^n$ be a descent direction of f in x^k.

Find the exact step-length t^k in direction d^k such that the decrease of f is maximal.

☐ Exercise 7

Consider the general descent method (Algorithmus 3.4) for the function

$$f : \mathbb{R} \to \mathbb{R}, \qquad f(x) = x^2$$

with starting point $x^0 := 1$ and the direction d^k and step-size t^k:

1. $d^k := -1$, $t^k := \left(\frac{1}{2}\right)^{k+2}$ with $k \in \mathbb{N}_0$,
2. $d^k := (-1)^{k+1}$, $t^k := 1 + \frac{3}{2^{k+2}}$ with $k \in \mathbb{N}_0$.

Verify that these choices of the parameters for $k \in \mathbb{N}_0$ lead to a decrease of the function f. In order to do that, present the sequence x^k generated by the Algorithmus 3.4 using induction with respect to k. Determine in each case $\lim_{x\to\infty} f(x^k)$ and compare them to the minimum of $f(x)$. Comment on the error.
