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Optimierung
3. Übungsblatt

☐ Exercise 8

Let $b \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n \times n}$.

Show that $x_0 \in \mathbb{R}^n$ is a minimal point of $\varphi : \mathbb{R}^n \to \mathbb{R},\ x \mapsto \varphi(x) := ||Ax - b||$ if and only if the *Gaussian normal equation* $A^*Ax_0 = A^*b$ holds.

Here, $A^* \in \mathbb{R}^{n \times n}$ denotes the transposed matrix to A, i.e. $\forall x, y \in \mathbb{R}^n : \langle Ax, y \rangle = \langle x, A^*y \rangle$.

Let $\mathcal{H}$ denote an $\mathbb{R}$-Hilbert space with scalar product $\langle \cdot, \cdot \rangle_{\mathcal{H}}$.

☐ Exercise 9

Let $b \in \mathcal{H}$ and $A \in \mathcal{L}_b(\mathcal{H}, \mathcal{H})$, the space of all linear, continous maps on $\mathcal{H}$.

Show that $x_0 \in \mathcal{H}$ is a minimal point of $\varphi : \mathcal{H} \to \mathbb{R},\ x \mapsto \varphi(x) := ||Ax - b||_{\mathcal{H}}$ if and only if the *Gaussian normal equation* $A^*Ax_0 = A^*b$ holds.

Here, $A^* \in \mathcal{L}_b(\mathcal{H}, \mathcal{H})$ denotes the adjoint operator to A, i.e. $\forall x, y \in \mathcal{H} : \langle Ax, y \rangle_{\mathcal{H}} = \langle x, A^*y \rangle_{\mathcal{H}}$.

☐ Exercise 10

Use the characterization given in Exercise 8 to solve the following linear regression problem:

Find a vector of parameters $x = (x_1, x_2) \in \mathbb{R}^2$ such that the corresponding regression line $\gamma_x : \mathbb{R} \to \mathbb{R}$, defined by $\gamma_x(t) := x_1 + tx_2$, approximates the following measuring points

t_i	1975	1980	1985	1990	1995
γ_i	30	35	38	42	44

optimally, i.e. such that x_1, x_2 solve the optimization problem:

$$(x_1, x_2) = \arg\min_{(y_1, y_2)} \sum_{i=1}^{5} (\gamma_i - \gamma_y(t_i))^2.$$

☐ Exercise 11

Let F be a nonempty, closed, convex subset of $\mathcal{H}$ and $x_0 \in \mathcal{H}$. Show that

$$\forall x \in F : \quad \Big\{ ||x_0 - x||_{\mathcal{H}} = \mathrm{dist}(x_0, F) \quad \Longrightarrow \quad \forall y \in F : \langle x_0 - x, y - x \rangle_{\mathcal{H}} \leq 0 \Big\}.$$