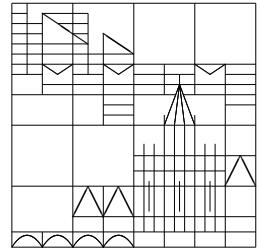


**Ausgabe:** 21.05.2012  
**Abgabe:** 29.05.2012, 11:00 Uhr, Briefkasten 11



## Optimierung 3. Übungsblatt

### □ Exercise 8

Let  $b \in \mathbb{R}^n$  and  $A \in \mathbb{R}^{n \times n}$ .

Show that  $x_0 \in \mathbb{R}^n$  is a minimal point of  $\varphi : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $x \mapsto \varphi(x) := \|Ax - b\|$  if and only if the *Gaussian normal equation*  $A^*Ax_0 = A^*b$  holds.

Here,  $A^* \in \mathbb{R}^{n \times n}$  denotes the transposed matrix to  $A$ , i.e.  $\forall x, y \in \mathbb{R}^n : \langle Ax, y \rangle = \langle x, A^*y \rangle$ .

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Let  $\mathcal{H}$  denote an  $\mathbb{R}$ -Hilbert space with scalar product  $\langle \cdot, \cdot \rangle_{\mathcal{H}}$ .

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### □ Exercise 9

Let  $b \in \mathcal{H}$  and  $A \in \mathcal{L}_b(\mathcal{H}, \mathcal{H})$ , the space of all linear, continuous maps on  $\mathcal{H}$ .

Show that  $x_0 \in \mathcal{H}$  is a minimal point of  $\varphi : \mathcal{H} \rightarrow \mathbb{R}$ ,  $x \mapsto \varphi(x) := \|Ax - b\|_{\mathcal{H}}$  if and only if the *Gaussian normal equation*  $A^*Ax_0 = A^*b$  holds.

Here,  $A^* \in \mathcal{L}_b(\mathcal{H}, \mathcal{H})$  denotes the adjoint operator to  $A$ , i.e.  $\forall x, y \in \mathcal{H} : \langle Ax, y \rangle_{\mathcal{H}} = \langle x, A^*y \rangle_{\mathcal{H}}$ .

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### □ Exercise 10

Use the characterization given in Exercise 8 to solve the following linear regression problem:

Find a vector of parameters  $x = (x_1, x_2) \in \mathbb{R}^2$  such that the corresponding regression line  $\gamma_x : \mathbb{R} \rightarrow \mathbb{R}$ , defined by  $\gamma_x(t) := x_1 + tx_2$ , approximates the following measuring points

$t_i$	1975	1980	1985	1990	1995
$\gamma_i$	30	35	38	42	44

optimally, i.e. such that  $x_1, x_2$  solve the optimization problem:

$$(x_1, x_2) = \arg \min_{(y_1, y_2)} \sum_{i=1}^5 (\gamma_i - \gamma_y(t_i))^2.$$


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### □ Exercise 11

Let  $F$  be a nonempty, closed, convex subset of  $\mathcal{H}$  and  $x_0 \in \mathcal{H}$ . Show that

$$\forall x \in F : \quad \left\{ \|x_0 - x\|_{\mathcal{H}} = \text{dist}(x_0, F) \quad \implies \quad \forall y \in F : \langle x_0 - x, y - x \rangle_{\mathcal{H}} \leq 0 \right\}.$$