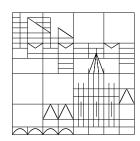
Universität Konstanz Fachbereich Mathematik und Statistik Prof. Dr. Michael Junk M. Gubisch, O. Lass, R. Mancini, S. Trenz Sommersemester 2012

**Ausgabe:** 18.06.2012

**Abgabe:** 25.06.2012, 11:00 Uhr, Briefkasten 11



## Optimierung 5. Übungsblatt

### □ Exercise 15

Let  $f : \mathbb{R} \to \mathbb{R}$  a four times differentiable, approximative function – which means that the function values f(x) are not known exactly, but with some error tolerance  $\epsilon_{\text{tol}}$ :

$$f_{\text{known}}(x) = f_{\text{exact}}(x) + \epsilon_{\text{tol}}(x)$$
 where  $|\epsilon_{\text{tol}}(x)| \le \epsilon$  for some known  $\epsilon > 0$ .

Determine the numerical first derivative  $D_c^1(f,h)$  by central differences and prove that the error  $\epsilon_H$  arising in the numerical approximation of the second derivative  $D_c^2(f,h)$  is of the order  $\epsilon_H = O(\epsilon^{\frac{4}{9}})$  for a suitable discretization stepsize h, i.e.

$$||D_c^2(f_{\text{known}}, h) - f_{\text{exact}}''||_{\infty} < \text{Const} \cdot \epsilon^{\frac{4}{9}}.$$

### □ Exercise 16

Let  $f \in \mathcal{C}^2(\mathbb{R}^n, \mathbb{R})$ . Verify the formula

$$\frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} f(x) = \frac{f(x + \epsilon e_i + \epsilon e_j) - f(x + \epsilon e_i) - f(x + \epsilon e_j) + f(x)}{\epsilon^2} + O(\epsilon)$$

for the approximation of the Hessian matrix by evaluations of f.

#### $\Box$ Exercise 17

Let  $f \in \mathcal{C}^1(\mathbb{R}^n, \mathbb{R})$  be a quadratic function of the form

$$f(x) = \frac{1}{2}\langle x, Qx \rangle + \langle c, x \rangle + \gamma$$
,  $Q \in \mathbb{R}^{n \times n}$  symmetric & positive definite,  $c \in \mathbb{R}^n$  and  $\gamma \in \mathbb{R}$ .

Let  $x_0 \in \mathbb{R}^n$  and H be a symmetric, positive definite matrix. Define  $\tilde{f}(x) := f(H^{-\frac{1}{2}}x)$  and  $\tilde{x}_0 = H^{\frac{1}{2}}x_0$ . Let  $(\tilde{x}_k)$  be generated by the Steepest Descent Method with optimal stepsize choice, i.e.  $\tilde{x}_{k+1} = \tilde{x}_k + \tilde{t}_k \tilde{d}_k$  where  $\tilde{d}_k = -\nabla \tilde{f}(\tilde{x}_k)$  and  $\tilde{t}_k = t(\tilde{d}_k)$  as determined in Exercise 6.

Let  $(x_k)$  generated by the gradient-like method  $x_{k+1} = x_k + t_k d_k$  with optimal stepsize  $t_k$  and preconditioner H, i.e.  $t_k = t(d_k)$  as in Exercise 6 and  $d_k = H^{-1}(-\nabla f(x_k))$ .

Show that the two optimization methods are equivalent, i.e.  $x^k = H^{-\frac{1}{2}}\tilde{x}_k$  holds for all k, and that the optimal stepsizes are identical:  $t_k = \tilde{t}_k$ .

# $\square$ Exercise 19 (UEFA Euro 2012)

Fill out the following table correctly:

	Competitor 1	Competitor 2	Result
Semifinal 1			:
Semifinal 2			:
Final			: