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Fachbereich Mathematik und Statistik
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Optimierung
6. Übungsblatt

☐ **Exercise 19** (Preconditioning)

Consider the quadratic function $f : \mathbb{R}^2 \to \mathbb{R}$,

$$f(x,y) = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 100 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 3,$$

with the conditioners

$$H = \mathrm{Id} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad H = \nabla^2 f = \begin{pmatrix} 100 & -1 \\ -1 & 2 \end{pmatrix}, \quad H = \begin{pmatrix} f_{xx} & 0 \\ 0 & f_{yy} \end{pmatrix} = \begin{pmatrix} 100 & 0 \\ 0 & 2 \end{pmatrix}.$$

Use the Gradient Method you implemented for the first program sheet on $\tilde{f}$ to determine the number of gradient steps required for finding the minimum of f with the different preconditionings.

☐ **Exercise 20** (Equality and inequality constraints)

Consider the constrained optimization problem

$$\max_{(x,y)} f(x,y) \qquad \text{subject to} \qquad (x,y) \in F$$

where

$$f(x,y) = x^2 + x^2y^2 + 9y^2 + 9, \qquad F = \{(a,b) \in \mathbb{R}^2 \mid 2a^4 + b^2 \leq 239\}.$$

1. Show that the problem has a global solution.
2. Draw the set of admissible points (you may use Matlab here).
3. Show that the problem has no inner solution and that boundary solutions cannot be unique.
4. Determine the corresponding Lagrange functional and solve the optimization problem.

☐ **Exercise 20** (Classical Newton method)

Consider the functions $f, g : \mathbb{R} \to \mathbb{R}$, given by

$$f(x) = x^3 - 2x + 2, \qquad g(x) = \sin(x).$$

1. Show that for the starting point $x_0 = 0$, the classical Newton iteration of f has two accumulation points which are both no zeros of f. Find another initial point which does not lead to a convergence of the Newton method applied on f.
2. Find a starting point x_0 such that the Newton iteration for g tends to $+\infty$.
3. Show why the methods do not converge to a zero of the functions by a suitable graphic.