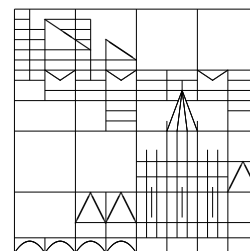


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## Optimierung 2. Program

### Implementation of the Projected Gradient Method.

**Part 1:** Generate a file `projection.m` and implement the function

```
function x = projection(x0, a, b)
```

with the current point  $x_0$ , lower bound  $a$  and upper bound  $b$ . The function returns the projected point  $x$  according to the projection

$$\mathbb{P} : \mathbb{R}^2 \rightarrow \Omega := \{x \in \mathbb{R}^2 \mid a_1 \leq x_1 \leq b_1, a_2 \leq x_2 \leq b_2\}.$$

defined as

$$\left(\mathbb{P}(x)\right)_i := \begin{cases} a_i & \text{if } x_i \leq a_i \\ x_i & \text{if } x_i \in [a_i, b_i] \\ b_i & \text{if } x_i \geq b_i. \end{cases}$$

Test your program by calculating  $\mathbb{P}(x_0)$  for a rectangular domain defined by the lower bound (lower left corner)  $a = (-1; -1)$ , the upper bound (upper right corner)  $b = (1; 1)$ , a direction  $d = (1.5; 1.5)$ , step sizes  $t = 0$  and  $t = 1$  and the following points  $x_0 = y + td \in \mathbb{R}^2$ :

Points $y$ :	Projections for $t = 0$	Projections for $t = 1$
(-2; -2)	(-1; -1)	(-0.5; -0.5)
(-1; -1)	(-1; -1)	(0.5; 0.5)
(-0.5; 0.5)	(-0.5; 0.5)	(1; 1)
(2; 0.5)	(1; 0.5)	(1; 1)
(1; -0.5)	(1; -0.5)	(1; 1)

Table 1: Testing points and their projections with respect to  $t$

**Part 2:** Write a function

```
function t = modarmijo(fhandle, x0, d, t0, alpha, beta, a, b)
```

for the Armijo step size strategy with the modified termination condition

$$f(x(\lambda)) - f(x) \leq -\frac{\alpha}{\lambda} \|x - x(\lambda)\|^2$$

with initial point  $x_0$ , descent direction  $d$ , initial step size  $t_0$ ,  $\alpha$  and  $\beta$  for the Armijo rule and  $a$  and  $b$  for the projection rule.

**Part 3:** Implement the gradient projection algorithm with direction  $d_k := -\frac{\nabla f(x_k)}{\|\nabla f(x_k)\|}$ . Generate a file `gradproj.m` for the function

```
function X = gradproj(fhandle, x0, epsilon, N, t0, alpha, beta, a, b)
```

with initial point `x0`, parameter `epsilon` for the termination condition  $\|\nabla f(x_k)\| < \epsilon$  and the additional termination condition  $\|x - x(1)\| < \epsilon$ , `N` for the maximal number of iteration steps, parameters `t0`, `alpha` and `beta` for the Armijo rule and `a` and `b` for the projection rule. Modify therefore the gradient method `gradmethod.m`.

The program should return a matrix `X = [x0; x1; x2; ...]` containing the whole iterations.

**Part 4:** Call the function `projgrad` from a main file `main.m` to test your program. Use the Rosenbrock function with the following parameters and write your observations in the report:

`epsilon=1.0e-2`, `N=1.5e+3`, `t0=1`, `alpha=1.0e-2`, `beta=0.5` and

1. `x0=[1;-0.5]`, `a=[-1;-1]` and `b=[2;2]`
2. `x0=[-1;-0.5]`, `a=[-2;-2]` and `b=[2;0]`
3. `x0=[-2;2]`, `a=[-2;-2]` and `b=[2;2]`

Visualize your results by suitable plots.