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Optimierung
2. Program

Implementation of the Projected Gradient Method.

Part 1: Generate a file `projection.m` and implement the function

```
function x = projection(x0, a, b)
```

with the current point `x0`, lower bound `a` and upper bound `b`. The function returns the projected point `x` according to the projection

$$\mathbb{P} : \mathbb{R}^2 \to \Omega := \{x \in \mathbb{R}^2 \mid a_1 \leq x_1 \leq b_1,\ a_2 \leq x_2 \leq b_2\}.$$

defined as

$$\Big(\mathbb{P}(x)\Big)_i := \begin{cases} a_i & \text{if } x_i \leq a_i \\ x_i & \text{if } x_i \in [a_i, b_i] \\ b_i & \text{if } x_i \geq b_i. \end{cases}$$

Test your program by calculating $\mathbb{P}(x_0)$ for a rectangular domain defined by the lower bound (lower left corner) $a = (-1; -1)$, the upper bound (upper right corner) $b = (1; 1)$, a direction $d = (1.5; 1.5)$, step sizes $t = 0$ and $t = 1$ and the following points $x_0 = y + td \in \mathbb{R}^2$:

Points y:	Projections for $t = 0$	Projections for $t = 1$
$(-2; -2)$	$(-1; -1)$	$(-0.5; -0.5)$
$(-1; -1)$	$(-1; -1)$	$(0.5; 0.5)$
$(-0.5; 0.5)$	$(-0.5; 0.5)$	$(1; 1)$
$(2; 0.5)$	$(1; 0.5)$	$(1; 1)$
$(1; -0.5)$	$(1; -0.5)$	$(1; 1)$

Table 1: Testing points and their projections with respect to t

Part 2: Write a function

```
function t = modarmijo(fhandle, x0, d, t0, alpha, beta, a, b)
```

for the Armijo step size strategy with the modified termination condition

$$f(x(\lambda)) - f(x) \leq -\frac{\alpha}{\lambda}||x - x(\lambda)||^2$$

with initial point `x0`, descent direction `d`, initial step size `t0`, `alpha` and `beta` for the Armijo rule and `a` and `b` for the projection rule.

Part 3: Implement the gradient projection algorithm with direction $d_k := -\frac{\nabla f(x_k)}{\|\nabla f(x_k)\|}$. Generate a file `gradproj.m` for the function

```
function X = gradproj(fhandle, x0, epsilon, N, t0, alpha, beta, a, b)
```

with initial point `x0`, parameter `epsilon` for the termination condition $\|\nabla f(x_k)\| < \epsilon$ and the additional termination condition $\|x - x(1)\| < \epsilon$, `N` for the maximal number of iteration steps, parameters `t0`, `alpha` and `beta` for the Armijo rule and `a` and `b` for the projection rule. Modify therefore the gradient method `gradmethod.m`.

The program should return a matrix `X = [x0; x1; x2; ...]` containing the whole iterations.

Part 4: Call the function `projgrad` from a main file `main.m` to test your program. Use the Rosenbrock function with the following parameters and write your observations in the report:

`epsilon=1.0e-2`, `N=1.5e+3`, `t0=1`, `alpha=1.0e-2`, `beta=0.5` and

1. `x0=[1;-0.5]`, `a=[-1;-1]` and `b=[2;2]`
2. `x0=[-1;-0.5]`, `a=[-2;-2]` and `b=[2;0]`
3. `x0=[-2;2]`, `a=[-2;-2]` and `b=[2;2]`

Visualize your results by suitable plots.