Universität Konstanz *SS 11*
Fachbereich Mathematik und Statistik
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Übungen zu **Numerische Verfahren der restringierten Optimierung**

http://www.math.uni-konstanz.de/numerik/personen/volkwein/teaching/

Submission: 22.06.2011, 12:00 o'clock
Codes by E-Mail and Reports together with the Homework

Program 3 (6 Points)

Given the nonlinear optimization problem

$$\min \frac{1}{2} x^\top Q x + x^\top d \quad \text{subject to} \quad Ax + F(x) = b,$$

where $A = [B|N]$ with B an invertible matrix and $F(x)$ nonlinear. Implement the SQP method to solve this problem following Algorithm 4.3. Name your function

```
[x,lambda] = mysqp(Q,d,A,b,Nonlin,x0,lambda0,tol,maxiter).
```

The input parameters `Q, d, A, b, x0, lambda0, maxiter` and `tol` are as in Program 1 and 2. The parameter `Nonlin` is a structure containing function handles `F`, `Fp` and `Fpp` in the form:

$$\texttt{Nonlin.F} = F(x), \quad \texttt{Nonlin.Fp} = \nabla F(x) \quad \text{and} \quad \texttt{Nonlin.Fpp} = \lambda^\top \nabla^2 F(x).$$

In case that `Fpp` is not given the program should utilize a damped BFGS updating of the form (4.16) to aproximate $\nabla_{xx} L(x, \lambda)$, where $B_0 = Q$ is used (*Hint*: Use the commands `fieldnames` and `ismember`). To solve the subproblem (4.5) use `myquadprog` from the previous programming exercise. Test your codes with the following settings:

$$Q = \begin{pmatrix} I & 0 \\ 0 & \nu I \end{pmatrix}, \quad d = \begin{pmatrix} -Iz \\ 0 \end{pmatrix}, \quad A = \begin{pmatrix} L & -I \end{pmatrix}, \quad b = 0, \quad F(x) = (x_1^3, \ldots, x_n^3)$$

with

$$L = \frac{1}{h^2} \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix} \in \mathbb{R}^{n \times n}, \quad I = \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix} \in \mathbb{R}^{n \times n}$$

and $z = \frac{1}{8} \sin(x)(17\nu - 60\nu \cos(2x) + 3\nu \cos(4x) + 8)$ for $x \in [h, 2\pi - h]$, where $h = 2\pi/(n+1)$. Further set $\nu = 10^{-4}$. As a stopping criteria for the SQP method choose $\|\Delta x\|_2 <$ `tol` with `tol` $= 10^{-6}$ and set `maxiter` $= 20$. Try different values for n (i.e. 50, 100, 500, 1000, 1500). Compare the performance of the SQP and the SQP-BFGS implementation. Don't forget to check the dimensions of the input arguments and inform the user if `maxiter` is reached. Document the code well, following the template given in Program 1. In the writen report give some details on the derivatives.