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Exercises for
Theory and Numerics of Partial Differential Equations

http://www.math.uni-konstanz.de/numerik/personen/mechelli/teaching.html

Sample Exercise

Exercise 1. (Matlab)
Implement Midpoint Rule, Trapezoidal Rule and Simpson Rule[1] for approximating:

$$\int_a^b f(x)dx \tag{1}$$

with $b > a > 0$, $x \in [a, b]$ and $f(x)$ a Riemann integrable function. The code has to be structured in the following way:

1. Each rule has to be implemented separately as a function `Rulename(x,Δx,f)` which take in input the vector of spatial nodes `x` generated with a spatial step Δ`x` and a function handle `f` to be integrated. The function `Rulename(x,dx,f)` has to give in output the approximated value of (1), computed with that rule.
2. In `main.m`, call each rule for the functions $f_1(x) = x(1 + x^2)$, $f_2(x) = \ln(x)$ and $f_3(x) = -2x\exp^{-x^2}$ for different values of Δ`x`, a and b, plotting the error between the exact and the approximated solution and verifying the convergence rate, and thus the order, of each method[2].

Write a report commenting the approximation errors for different values of Δ`x` and the order of each method applied to $f_1(x)$,$f_2(x)$ and $f_3(x)$.

[1] In Germany, it is most known as Keplersche Fassregel.

[2] Fix an initial value of a, b and Δ`x`, and then compute in a for loop the error between the exact solution I_{ex} and the approximated one I_n, dividing Δ`x` by 2 at each loop iteration. The decay of the error in a semi-logarithmic plot should have the same slope of Δx^q, where q is the method's order; use the command `semilogy` for instance. This order can be also computed analytically, it equals to $\log_2(\frac{|I_{n-1}-I_{ex}|}{|I_n-I_{ex}|})$. Be aware that for small values of Δ`x` you can have round-off errors.