University of Konstanz
Department Mathematics and Statistics
J. Lu, L. Mechelli, Prof. Dr. S. Volkwein

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Exercises for
Theory and Numerics of Partial Differential Equations

http://www.math.uni-konstanz.de/numerik/personen/mechelli/teaching.html

Sample Report

These numerical tests have been made on a Notebook Lenovo ThinkPad T450s with Intel Core i7-5600U CPU @ 2.60GHz and 12GB RAM[1]. We know from the theory that the Midpoint Rule and the Trapezoidal Rule are 2nd order methods and the Simpson Rule is of the 4th order. We are expecting that the numerical simulations reflect these behaviours. For these tests we have chosen $[a, b] = [0.5, 2.5]$. In Table 1 the results for the Midpoint Rule, used for solving the integral of $f_1(x) = x(1 + x^2)$, are shown: as expected we have that the

Iteration n	Δx	$\lvert I_n - I_e x\rvert$	Order
1	0.5	7.5000×10^{-3}	–
2	0.25	1.8750×10^{-3}	2.00
3	0.125	4.6875×10^{-4}	2.00
4	0.0625	1.1719×10^{-4}	2.00
5	0.03125	2.9297×10^{-5}	2.00
6	0.015625	7.3242×10^{-6}	2.00

Table 1: Error values for Midpoint Rule applied to integral of $f_1(x)$

convergence rate[2] is 2. This can also be seen in Figure 1(a), where the plot of the error in a semi-logarithmic scale of each integral approximation has the same slope of $(\frac{\Delta x}{2^{n-1}})^2$, where the initial $\Delta x = 0.5$ and n is the respective iteration of the for-loop. We have the same behaviour[3] for the integral of $f_2(x)$ and $f_3(x)$. Also for

Iteration n	Δx	$\lvert I_n - I_e x\rvert$	Order
1	0.5	1.5000×10^{-2}	–
2	0.25	3.7500×10^{-3}	2.00
3	0.125	9.3750×10^{-4}	2.00
4	0.0625	2.3437×10^{-4}	2.00
5	0.03125	5.8594×10^{-5}	2.00
6	0.015625	1.4648×10^{-5}	2.00

Table 2: Error values for Trapezoidal Rule applied to integral of $f_1(x)$

the Trapezoidal rule, the numerical tests confirm what we expected, as one can see in Table 2 and in Figure 1(b). We have decided to report still the values for the integral of $f_1(x)$ because it is not a coincidence that the error of the Trapezoidal Rule is exactly the double of the one of Midpoint Rule: it can be shown that this property

Iteration n	Δx	$\lvert I_n - I_e x\rvert$	Order
1	0.5	4.2266×10^{-9}	–
2	0.25	2.7478×10^{-10}	3.94
3	0.125	1.7338×10^{-11}	3.99
4	0.0625	1.0862×10^{-12}	4.00
5	0.03125	6.7502×10^{-14}	4.01
6	0.015625	5.2180×10^{-15}	3.69

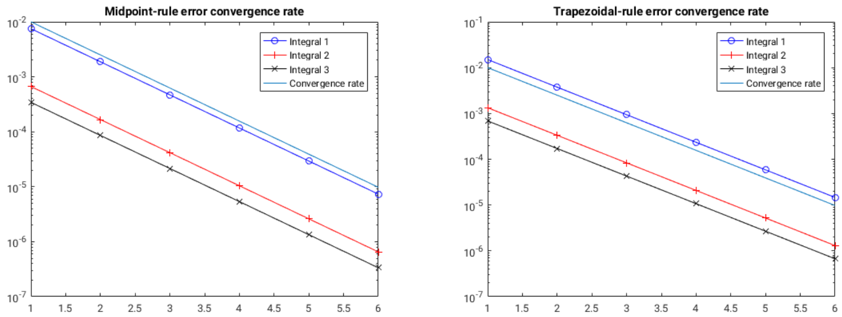
Table 3: Error values for Simpson Rule applied to integral of $f_3(x)$

[1] It is always good to specify CPU and RAM of your PC, in particular when you report the computational time.

[2] We know from the theory that the convergence rate is an asymptotic behaviour, so it should be verified for $\Delta x \to 0$. When we ask to do that in the exercises, it is fine to provide the results for 5-6 different decreasing Δx.

[3] To save time, we do not report in a table the errors' data for the approximation of $f_2(x)$ and $f_3(x)$, but in the exercises you have to provide all the data we asked for. Do not worry, we will ask only things that can be reported in a reasonable time.

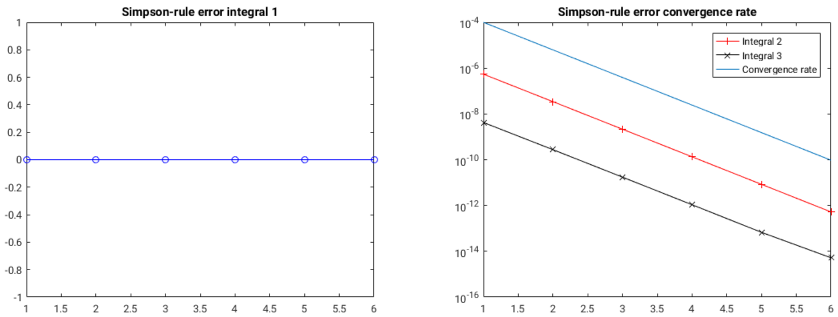
holds for the approximation of the integral of every polynomial of degree 3 or less. Other interesting behaviours appear with the Simpson Rule that is a method of the 4th order. First of all, we can notice from Figure 2(a) that the integral of $f_1(x)$ is exactly approximated by the method for every value of Δx, i.e. the error between the exact solution and the approximated one is always zero. This is already predicted by the theory, since



(a) Midpoint rule errors (b) Trapezoidal rule errors

Figure 1: Plot of errors for Midpoint-rule and Trapezoidal-rule

the Simpson Rule is a method of the 4th order, we can approximate the integral of a polynomial of degree 3 (or less)[4] in an exact way. As shown in Figure 2(b) and in Table 3, the numerical tests show the expected convergence rate for the Simpson Rule. In the last line of Table 3, the order is 3.69, so it seems to decrease, but this is due to the fact that the error is close to the machine precision, so the logarithmic formula does not give anymore significant values.



(a) Simpson rule error for $f_1(x)$ (b) Simpson rule errors for $f_2(x)$ and $f_3(x)$

Figure 2: Plot of errors for Midpoint-rule and Trapezoidal-rule

[4]This is because we have an error bound for the method that depends on the forth derivative of $f(x)$.