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Numerical Methods for Partial Differential Equations
Tutorial 10

Exercise 19 (High resolution schemes; programming exercise):

High resolution schemes are designed to satisfy the TVD property for scalar conservation laws (and systems of equations with constant coefficients), whereby spurious oscillations characteristic to high order schemes are supressed. Solve the initial value problem for the linear advection equation $\frac{\partial u}{\partial t} + a\,\frac{\partial u}{\partial x} = 0$ in $\mathbb{R} \times (0,\infty)$, $a > 0$ by using the *modified flux approach* in the form $v_i^0 := \int_{x_{i-1/2}}^{x_{i+1/2}} u_0(x)\,dx,\ i \in \mathbb{Z}$,

$$\begin{aligned} v_i^{n+1} &= v_i^n - \lambda\, a(v_i^n - v_{i-1}^n) \\ &\quad - \frac{\lambda a(1-\lambda\, a)}{2}\left[\varphi\left(\frac{v_i^n - v_{i-1}^n}{v_{i+1}^n - v_i^n}\right)(v_{i+1}^n - v_i^n) - \varphi\left(\frac{v_{i-1}^n - v_{i-2}^n}{v_i^n - v_{i-1}^n}\right)(v_i^n - v_{i-1}^n)\right],\ i \in \mathbb{Z},\ n = 0, 1, \ldots \end{aligned}$$

with the following *flux limiters*:

1) The “minmod” flux limiter:

$$\varphi_1(r) := \max\Big\{0,\ \min\{r, 1\}\Big\};$$

2) The “superbee” flux limiter of Roe:

$$\varphi_2(r) := \max\Big\{0,\ \min\{1,\ 2r\},\ \min\{r,\ 2\}\Big\};$$

3) The flux limiter by van Leer:

$$\varphi_3(r) := \frac{r + |r|}{1 + |r|}, \quad r \in \mathbb{R};$$

4) The flux limiter by Osher and Chakravarthy:

$$\varphi_4(r) := \max\Big\{0,\ \min\{r, \alpha\}\Big\}, \quad \alpha \in [1, 2].$$