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WS 2001/02

Numerical Methods for Partial Differential Equations
Tutorial 9

Exercise 15:

Let $v_i^{n+1} = \mathcal{H}(v_{i-\ell}^n, \dots, v_i^n, \dots, v_{i+\ell}^n)$, $i \in \mathbb{Z}$, $n = 1, 2, \dots$ be a finite difference scheme which can be written in the *incremental form*

$$v_i^{n+1} = v_i^n + C(v_{i-\ell+1}^n, \dots, v_{i+\ell}^n) \cdot (v_{i+1}^n - v_i^n) - D(v_{i-\ell}^n, \dots, v_{i+\ell-1}^n) \cdot (v_i^n - v_{i-1}^n),$$

with bounded functions $C, D : \mathbb{R}^{2\ell} \to \mathbb{R}$. Prove that, if the incremental coefficients

$$C_{i+1/2}^n := C(v_{i-\ell+1}^n, \dots, v_{i+\ell}^n), \quad D_{i-1/2}^n := D(v_{i-\ell}^n, \dots, v_{i+\ell-1}^n)$$

satisfy the relations

$$C_{i+1/2}^n, D_{i+1/2}^n \geq 0 \quad \text{and} \quad C_{i+1/2}^n + D_{i+1/2}^n \leq 1 \quad \text{for all} \quad i \in \mathbb{Z},\ n = 1, 2, \dots,$$

then the scheme is TVD.

Exercise 16:

Write the Lax–Friedrichs method in incremental form and show herewith that the method is L^∞–stable and TVD, in the case the CFL–condition is satisfied.

Exercise 17:

Show that any ℓ^1–contracting numerical method is TVD.

Exercise 18:

Show that the upwind schemes and the Lax–Friedrichs scheme are ℓ^1–contracting, provided the CFL–condition is satisfied.