Tutorial CFD
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Tutorial 10

To be discussed on: Monday, 21.07.
Download: www.mathematik.uni-kl.de/~rutka/CFD_tutorial.html

1. Finite Elements

Recall the definitions:

1. A triplet (K, P, Σ), where

   K is a domain,

   P: a vector space of polynomials defined on K with some dimension N_p,

   $\Sigma := \{L_1, L_2, \ldots L_{N_p}\}$: a set of linear functionals $L_i : P \longrightarrow \mathbf{R}$

   is called a **finite element**.

2. A finite element is called **unisolvant** if $\forall g \in P$ we have $L_1(g) = L_2(g) = \ldots = L_{N_p} = 0 \Rightarrow g \equiv 0$.

3. Let (K, P, Σ) be a finite element. We say that a set of functions $\{\Theta_1, \Theta_2, \ldots \Theta_{N_p}\}$ has the **δ-property** if $L_i(\Theta_j) = \delta_{ij}$.

Consider the following examples:

(A) $P := \{ax + b \,|\, a, b \in \mathbf{R}\}$,
$L_1(g) = g(x_i)$, $L_2(g) = g(x_{i+1})$.

(B) $P = \{ax^2 + bx + c \,|\, a, b, c \in \mathbf{R}\}$,
$L_1(g) = g(x_i)$, $L_2(g) = g(x_{i+1})$, $L_3(g) = \partial_x g(x_i)$.

(C) $P = \{ax^2 + bx + c \,|\, a, b, c \in \mathbf{R}\}$,
$L_1(g) = g(x_i)$, $L_2(g) = g(x_{i+1})$, $L_3(g) = 2g(x_i)$.

K is an interval $[x_m, x_{m+1}]$ in all cases.

Check the unisolvancy of these examples. Construct the set of functions Θ satisfying the δ-property if possible.

2. Stream and Vorticity functions

2.1

Check that the stream function-vorticity formulation of the (non-dimensional) Navier Stokes equations

$$\begin{aligned} \frac{\partial \underline{u}}{\partial t} + (\underline{u} \cdot \nabla)\underline{u} &= -\nabla p + \frac{1}{Re}\nabla^2 \underline{v} \\ \nabla \cdot \underline{u} &= 0 \end{aligned}$$

is

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{Re}\nabla^2 \omega\,,$$

where $\underline{u} = (u, v)$, Ψ is the stream function and ω is the vorticity.

2.2

Given the stream function, calculate and plot the velocity and vorticity.

(A) $\Psi = \sin x + \cos y$

(B) $\Psi = x^5 + xy + y^4$