Tutorial CFD
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Tutorial 5

To be discussed on: Monday, 16.06.
Download: www.mathematik.uni-kl.de/˜rutka/CFD_tutorial.html

Programming Exercise

Implement the *Godunov's scheme*

$$u_i^{n+1} = u_i^n - \lambda \left(f(u^*(u_i^n, u_{i+1}^n)) - f(u^*(u_{i-1}^n, u_i^n)) \right)$$

where $u^*(u_L, u_R)$ is given by

$f'(u_L)$	$f'(u_R)$	value of u^*
≥ 0	≥ 0	u_L
≥ 0	< 0	u_L if $s \geq 0$ u_R if $s < 0$
< 0	< 0	u_R
< 0	≥ 0	u_{sonic}

$s = \frac{f(u_R)-f(u_L)}{u_R-u_L}$: the shock speed
u_{sonic}: the sonic point, characterized by $f'(u_{sonic}) = 0$.
$\lambda = k/h$.

Consider the following examples:

A) $f(u) = u^2/2$ (Burger's equation)

B) $f(u) = u^4$

C) $f(u) = u$ (linear advection with speed 1)

Try the following initial conditions:

init.A) $u_0(x) = \begin{cases} -2 & , \quad x < 1 \\ 1 & , \quad x \geq 1 \end{cases}$

init.B) $u_0(x) = \begin{cases} 2 & , \quad x < 0 \\ -1 & , \quad x \geq 0 \end{cases}$

init.C) $u_0 = \sin(x), \quad x \in [0, 10]$

Note: there is no assumption about the periodicity. How do you treat the boundary nodes?

Choose a proper time step, so that the CFL condition is satisfied.