Tutorial CFD
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Tutorial 6

To be discussed on: Monday, 23.06.
Download: www.mathematik.uni-kl.de/~rutka/CFD_tutorial.html

Programming Exercise: Euler Equations

A flow of an inviscid, compressible gas in a tube is described by 1D compressible Euler equations:

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho u\\ E\end{pmatrix}+\frac{\partial}{\partial x}\begin{pmatrix}\rho u\\ \rho u^2+p\\ (E+p)u\end{pmatrix}=0 \tag{1}$$

where ρ is the density, u: velocity, p: the pressure and E is the internal energy. Relation between E and p is given by

$$E=\frac{p}{\gamma-1}+\frac{1}{2}\rho u^2\,.$$

For a diatomic gas (e.g., air), $\gamma=7/5$.

The Lax-Friedrich's schema to d-dimensions is straight-forward extension of the one-dimensional case. For the conservation law

$$\frac{\partial \mathbf{w}}{\partial t}+\frac{\partial \mathbf{f}(\mathbf{w})}{\partial x}=\mathbf{0}$$

with some $\mathbf{w}\in\Re^d$ and $\mathbf{f}:\Re^d\to\Re^d$, it is given by

$$\mathbf{w}_i^{n+1}=\frac{1}{2}\left(\mathbf{w}_{i+1}^n+\mathbf{w}_{i-1}^n\right)-\frac{\lambda}{2}\left(\mathbf{f}(\mathbf{w}_{i+1}^n)-\mathbf{f}(\mathbf{w}_{i-1}^n)\right)\,.$$

$\lambda=\tau/h$, where τ and h are time and space steps respectively.

Implement the Lax-Friedrich's scheme for the equations (1)

1. using the *primitive variables* ρ, u and p
2. using the *conservative variables* $q_1=\rho$, $q_2=\rho u$, $q_3=E$.

Consider the following initial data:

1. $\rho^0=5$ if $x<0$, $\rho^0=1$ otherwise,
   $u^0=0$, $p^0=0$.
2. $\rho^0=1$, $u^0=0$,
   $p^0=0$ if $x<0$, $p^0=5$ otherwise.
3. $\rho^0=5$ if $x<0$, $\rho^0=1$ otherwise,
   $u^0=5$ if $x<0$, $u^0=0$ otherwise,
   $p^0=0$.

Compare the results which you get by using primitive and conservative variables. What is the physical interpretation of the given initial conditions?