TABLE 2.2: Data for the outer solar system

planet	mass	initial position	initial velocity
Jupiter	$m_1 = 0.000954786104043$	−3.5023653 −3.8169847 −1.5507963	0.00565429 −0.00412490 −0.00190589
Saturn	$m_2 = 0.000285583733151$	9.0755314 −3.0458353 −1.6483708	0.00168318 0.00483525 0.00192462
Uranus	$m_3 = 0.0000437273164546$	8.3101420 −16.2901086 −7.2521278	0.00354178 0.00137102 0.00055029
Neptune	$m_4 = 0.0000517759138449$	11.4707666 −25.7294829 −10.8169456	0.00288930 0.00114527 0.00039677
Pluto	$m_5 = 1/(1.3 \cdot 10^8)$	−15.5387357 −25.2225594 −3.1902382	0.00276725 −0.00170702 −0.00136504

Outer Solar System We next apply our methods to the system which describes the motion of the five outer planets relative to the sun. This system has been studied extensively by astronomers, who integrated it for a time span of nearly 100 million years and concluded the chaotic evolution of the solar system [SW92]. The problem is a Hamiltonian system (2.2) with

$$H(p,q) = \frac{1}{2}\sum_{i=0}^{5} m_i^{-1} p_i^T p_i - G \sum_{i=1}^{5}\sum_{j=0}^{i-1} \frac{m_i m_j}{\|q_i - q_j\|}. \tag{2.10}$$

Here p and q are the supervectors composed by the vectors $p_i, q_i \in \mathbb{R}^3$ (momenta and positions), respectively. The chosen units are: masses relative to the sun, so that the sun has mass 1. We have taken

$$m_0 = 1.00000597682$$

in order to take account of the inner planets. Distances are in astronomical units (1 [A.U.] = 149 597 870 [km]), times in earth days, and the gravitational constant is

$$G = 2.95912208286 \cdot 10^{-4}.$$

The initial values for the sun are taken as $q_0(0) = (0,0,0)^T$ and $\dot{q}_0(0) = (0,0,0)^T$. All other data (masses of the planets and the initial positions and initial velocities) are given in Table 2.2. The initial data are taken from "Ahnerts Kalender für Sternfreunde 1994", Johann Ambrosius Barth Verlag 1993, and they correspond to September 5, 1994 at 0h00.[1]

To this system we applied our four methods, all with step size $h = 10$ (days) and over a time period of 200 000 days. The numerical solution (see Fig. 2.3) behaves similarly to

[1] We thank Alexander Ostermann, who provided us with all these data.