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Tutorial *Dynamical Systems: Theory and Numerics*
Return: 12am, Thursday, 15.11. metal box: Rutka/Schick (48, ground floor)

Exercise 1: *(Bernoulli's equation)*

The differential equation $y'+g(x)y+h(x)y^\alpha = 0$ with $\alpha \neq 1$ is called *Bernoulli's equation.*

Determine how to solve such an equation (literature!) and apply this approach to

$$y' = x^4 y + x^4 y^4, \quad y(0) = \eta.$$

Exercise 2:

Solve the following system of ODE's:

$$\begin{aligned} \dot{x} &= -y + x(1 - x^2 - y^2) \\ \dot{y} &= x + y(1 - x^2 - y^2) \end{aligned}$$

(Hint: Use polar coordinates!)

Plot the trajectories $(x(t), y(t))$ for $x(0) = k \cdot h$, $y(0) = 0$ with $h = \frac{1}{4}$ and $k = 0, 1, \dots, 8$ and for $t \in [0, 4\pi]$.

Exercise 3: *(Discrete population growth)*

A discrete population model is given by

$$p_{n+1} = r p_n (1 - p_n), \qquad (*)$$

where $p_0 \in [0, 1]$ is given.

a) Determine all fixed points of this equation and analyze their stability. (Hint: Linearize around fixed points.)

b) For which $r > 0$ exist 2-cycles, i.e. $p_{n+2} = p_n \neq p_{n+1}$?

Exercise 4: *(Programming exercise)*

Consider the population model from exercise 3. Plot the limit points of the sequence p_n given by $(*)$ depending on $r \in [0, 4]$. Use $p_0 = \frac{1}{3}$.