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**Tutorial No. 4** ***Dynamical Systems: Theory and Numerics***

Return: 12am, Thursday, 6.12. metal box: Rutka/Schick (48, ground floor)

**Exercise 13:**

Compute the phase portrait of the system

$$\begin{aligned}\dot{x} &= x - 2y\\ \dot{y} &= 3x - 4y\end{aligned}$$

**Exercise 14:** *(Mathematical pendulum)*

The mathematical pendulum with friction is given by

$$\ddot{x} + f(x)\dot{x} + \sin x = 0,$$

where $f \in \mathcal{C}^1$ and $f > 0$ (damping). Show that no periodic solutions can exist.

**Exercise 15:**

Show that the system

$$\begin{aligned}\dot{x} &= x + y - x(x^2 + y^2)\\ \dot{y} &= -x + y - y(x^2 + y^2)\\ \dot{z} &= -z\end{aligned}$$

has a limit cycle $x^2 + y^2 = 1$, $z = 0$. Show by linear approximation that the origin is unstable. Show that the limit cycle is stable using cylindrical coordinates.

**Exercise 16:** *(An epidemic model)*

An epidemic model is given as follows. Let a population be divided into three categories: People who are susceptible to a disease ($x$), those who are infected ($y$), and those who are immune ($z$). The model is described by the following equations:

$$\begin{aligned}\dot{x} &= -\beta xy + \mu\\ \dot{y} &= \beta xy - \gamma y\\ \dot{z} &= \gamma y - \mu\end{aligned}$$

Here $\beta$, $\gamma$ and $\mu$ are positive constants and we only consider positive populations $x, y, z \geq 0$. Show that the overall population is constant. Determine the equilibrium solutions and analyze the behaviour in their neighbourhood. Discuss the model.