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Tutorial No. 9 ***Dynamical Systems: Theory and Numerics***
Return: 12am, Thursday, 31.01.02

Exercise 32:

a) Prove the following:

   If an implicit RK-method with non-singular A satisfies one of the conditions

   i) $\exists k \in \{1, \dots, s\}$ such that $a_{kj} = b_j \quad \forall j = 1, \dots, s$ **or**
   ii) $\exists k \in \{1, \dots, s\}$ such that $a_{ik} = b_k \quad \forall i = 1, \dots, s$

   then $R(\infty) = 0$. Thus an A-stable method satisfying i) or ii) is L-stable.

b) Why does the implicit trapezoidal method not contradict the above statement?

Exercise 33:

Consider

$$\dot{x} = \lambda(1 - x^2), \quad \lambda > 0, \ x(0) = 2.$$

a) Compute the analytic solution for $\lambda = 1$ (not with Maple).

b) Apply the implicit Euler method: Do one implicit step by hand. What happens? What does the Newton method do in this situation?

Exercise 34: *(Collocation methods)*

Given $\dot{x} = f(t,x)$ and a vector $c = (c_1, \dots, c_s)$ with $0 \le c_1 < c_2 < \dots < c_s \le 1$. In order to construct a polynomial $u \in \mathcal{P}_s$ satisfying the above system at the s collocation points $t + c_i h$ for $h > 0$ consider

$$\begin{aligned} u(t) &= x \\ \dot{u}(t + c_i h) &= f(t + c_i h, u(t + c_i h)), \quad i = 1, \dots, s \end{aligned}$$

a) Prove that

$$u(t + c_i h) = x + h \sum_{j=1}^{s} a_{ij} k_i$$

where $k_i = \dot{u}(t + c_i h)$, $a_{ij} = \int\limits_0^{c_i} L_j(\tau) \mathrm{d}\tau$ and $L_1, \dots, L_s$ form the Lagrange-basis of $\mathcal{P}_{s-1}$ wrt. the nodes $c_1, \dots, c_s$.

b) Show that the above construction leads to an IRK $\begin{array}{c|c} c & A \\ \hline & b \end{array}$ and compute b.

c) Consider the nodes of the Gaussian quadrature formulas of order $p = 2$ $(c_1 = \frac{1}{2})$ and $p = 4$ $(c_1 = \frac{1}{2} - \frac{\sqrt{3}}{6}, c_2 = \frac{1}{2} + \frac{\sqrt{3}}{6})$ and compute the resulting IRK's.