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Tutorial No. 10 ***Dynamical Systems: Theory and Numerics***
Return: 12am, Thursday, 7.2.02

Exercise 35:

Prove the first lemma from the lecture concerning collocation methods. ("The above defined IRK-scheme is consistent ... ")

Exercise 36:

Prove the proposition following the lemma of exercise 35, i.e. that

$$|\varphi^t x - \varphi^t y| \leq |x - y| \Leftrightarrow f \text{ dissipative}$$

Exercise 37:

Prove the theorem from the lecture concerning conservation of quadratic first integrals by Gauß-methods, which states $\mathcal{E}(\Psi^h x) = \mathcal{E}(x)$.

Exercise 38: *(Programming exercise)*

Given the Predator-Prey model

$$\begin{aligned} \dot{x} &= ax - cxy \\ \dot{y} &= -by + dxy. \end{aligned}$$

a) Compute a first integral of the system.

b) Use $a = 1,\ b = 2,\ c = 1,\ d = 1,\ (u_0, v_0) = (2, 5)$ and $h = \frac{1}{8}$ and apply an explicit solver of your choice, implicit Euler, symplectic Euler and Gauß-method with 1-stage. Compare the results, esp. the values of the first integral.