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Tutorial ***Dynamical Systems: Theory and Numerics***

Return: 12am, Thursday, 14.02. metal box: Rutka/Schick (48, ground floor)

Exercise 38:

Let $\lambda_1, \dots \lambda_l \in \mathbb{C}$ be the zeros of the characteristic polynomial ρ of a linear homogeneous difference equation of order k. Let $m_1 \dots m_l$ be the multiplicities of these zeros. The following are equivalent:

i) u is a solution of the difference equation,

ii) $u = (u_n)_{n \in \mathbb{N}}$,

$$u_n = \sum_{i=1}^{l} p_i(n) \lambda_i^n \,, \; n \in \mathbb{N}, \; p_i \in \mathcal{P}_{m_i - 1} \,.$$

a) Proof. Hint: write the difference equation in the form

$$U_n = \mathbf{A} U_{n-1} \,, \text{ where } U_n = \begin{pmatrix} u_n \\ \vdots \\ u_{n+k-1} \end{pmatrix} \in \mathbb{C}^k \text{ and } \mathbf{A} \in \mathbb{C}^{k \times k} \,.$$

b) Solve

$$u_{n+3} - 4u_{n+2} - 5u_{n+1} - 2u_n = 0$$

with initial values $u_0 = 1$, $u_1 = 2$, $u_2 = -1$.

Exercise 39:

Proof Dahlquist's root condition.

Exercise 40:

Proof: The following are equivalent:

a) A k-step method is consistent,

b) 1) $u_i \xrightarrow{h \to 0} x(t_0)$, $i = 0, \dots, k-1$,

2) $\rho(1) x(t) = 0$,

3) $\Psi(t, x(t), x(t+h), \dots, x(t+kh), h) - \rho'(1) f(t, x(t)) \xrightarrow{h \to 0} 0$.

Exercise 41:

Consider the 2-step method

$$u_{i+1} - 3u_i + 2u_{i-1} = -hf_i \,.$$

a) Show, that the method is consistent, provided u_0, u_1 are consistent. Order?

b) Try to solve the problem

$$\dot{x} = -x \;, \quad x(0) = 1 \,.$$

Does the method converge for all choices of

$$u_1 = \alpha(h) \;, \quad u_1 \xrightarrow{h \to 0} 1 \;?$$

Generate a value for u_1 using the improved Euler. What happens?